Abstract

In this thesis we study some generalized shock models with applications in optimal maintenance. Most of the systems used in reality are directly or indirectly affected by some harmful "instantaneous" events (shocks of different nature), which either cause the system's failure or decrease the system's lifetime. Thus, the study of systems' lifetimes subject to external shocks is one of the important problems in reliability theory. In a shock modeling, one has to answer two important questions, namely, "how the occurrences of shocks affect or decrease the lifetime of a system" and "how one can model the occurrences of shocks on a system ". In this thesis, we answer these questions in different setups.

Existing shock models are usually classified into four broad classes, namely, extreme shock models, cumulative shock models, run shock models and δ-shock models. The δ-shock model, which is an object of our study, is different in nature from other aforementioned shock models. In all δ-shock models developed so far in the literature, the recovery time δ was assumed to be constant. However, this assumption is too restrictive and unrealistic in describing many real-life scenarios. Indeed, δ can obviously depend on other parameters, namely, magnitude of shocks, arrival times of shocks, etc. This motivates us to introduce a new *time-dependent δ-shock model* wherein the recovery time of a system is assumed to be an increasing function of arrival times of shocks. For this model, we assume that shocks occur according to the generalized Pólya process (GPP) that contains the homogeneous Poisson process (HPP), the non-homogeneous Poisson process (NHPP) and the Pólya process as particular cases. We further generalize this model to the *general δ-shock model* by considering the recovery time δ as the function of both arrival times and magnitudes of shocks. We also consider a more general and flexible shock process, namely, the Poisson generalized gamma process (PGGP) that includes the HPP, the NHPP, the Pólya process and the GPP as the particular cases. With the same motivation, we study a *history-dependent mixed shock model* which is a combination of the history-dependent extreme shock model and the history-dependent δ-shock model. As an application of the aforementioned new shock models, we study the optimal replacement policy.

Although Poisson processes are widely used in various applications for modeling of recurrent point events, there exist obvious limitations. Several specific mixed Poisson processes (which are formally not Poisson processes any more) that were recently introduced in the literature overcome some of these limitations. We define a general mixed Poisson

process with the phase-type (PH) distribution as the mixing one. As the PH distribution is dense in the set of lifetime distributions, the new process can be used to approximate any mixed Poisson process. We study some basic stochastic properties of this new process and discuss some relevant applications by considering the extreme shock model, the stochastic failure rate model and the δ-shock model.

We introduce and study a general class of shock models with dependent inter-arrival times of shocks that occur according to the homogeneous Poisson generalized gamma process (HPGGP). A lifetime of a system affected by a shock process from this class is represented by the convolution of inter-arrival times of shocks. This class contains many popular shock models, namely, the extreme shock model, the generalized extreme shock model, the run shock model, the generalized run shock model, specific mixed shock models, etc. For systems operating under shocks, we derive and discuss the main reliability characteristics and illustrate our findings by the application that considers an optimal mission duration policy.

Counting processes based on heavy-tailed distributions (namely, the fractional homogeneous Poisson process (FHPP), the renewal process of matrix Mittag-Leffler type (RPMML), etc.) have not yet been considered in the literature for modeling the occurrences of shocks. Thus, we study some general shock models under the assumption that shocks occur according to a renewal process with the matrix Mittag-Leffler (MML) distributed inter-arrival times. As the class of MML distributions is wide and well-suited for modeling the heavy tail phenomena, these shock models can be very useful for analysis of lifetimes of systems subject to random shocks with inter-arrival times having heavier tails. Some relevant stochastic properties of the introduced models are described. Moreover, two applications, namely, the optimal replacement policy and the optimal mission duration are discussed.

Lastly, we consider coherent systems subject to random shocks that can damage a random number of components of a system. Based on the distribution of the number of failed components, we discuss three models, namely, (*i*) a shock can damage any number of components (including zero) with the same probability, (*ii*) each shock damages, at least, one component, and (*iii*) a shock can damage, at most, one component. Moreover, the arrivals of shocks are modeled using three important counting processes, namely, the PGPP, the Poisson phase-type process (PPHP) and the RPMML. For the defined shock models, we study some reliability properties of coherent systems. At the end, we discuss the optimal replacement policy as an application of the proposed models.