CHAPTER 1
Introduction

"*Live to learn, and you will really learn to live.*"

John C. Maxwell

For centuries, the study of light has been under discussion. Over the last 40 years, optics-photonics has become the dominant field covering areas like medicine, telecommunications, sensing systems, military technology, agriculture, etc. One of the most recent developments in this direction is using optical circuits and optical signal processing, leading to high-speed transmission and lower power consumption. Optic-photonics is an interdisciplinary field, bringing together researchers from diverse fields such as physics, engineering, materials science, biology, and mathematics.

Optics is the branch of physics concerned with the propagation of light, its behavior, its interaction with matter, and the construction of the instruments that use or detect it [Ghatak, 2017]. Modern optics is a subset of optics that explores the electromagnetic or quantum properties of light with advanced technology, leading to the development of the Internet, telecommunications, etc.

Photonics is an interconnected field with modern optics that deals with light in the form of photons, properties, and transmission [Okamoto, 2006]. This plays a vital role in several applications, such as practical photonic devices, health care, security and safety, optical data communications, etc. Such contributions help us to shape or develop 21th centuries in advance, such as optical computers, autonomous vehicles, lighting, and screens in paper form, which are just a few examples of the world of tomorrow.

Optics-photonics based technologies are vigorously explored as the next step to micro-electronics based technologies. Many research groups worldwide are working on various aspects of all-optical circuits and all-optical signal processing [Cheng, 1989]. Optical waveguides are one of the essential circuit components of such optical circuits [Kawano and Kitoh, 2004; Okamoto, 2006]. The research presented in this thesis is about the mathematical analysis of light propagation in bent waveguides. In this chapter, our focus will be on the basic concepts of the optical waveguides theory.

Brief orientation about the content of this chapter: In Sec. 1.1, Maxwell's equations are discussed briefly, including the dielectric medium waves. We discussed the waveguides and their classifications in Sec. 1.2. The wave propagation based on transverse electric (TE) and transverse magnetic (TM) are also discussed in this section. The straight and bent waveguides with their model description are covered in Sec. 1.3 and Sec. 1.4, respectively. The detailed literature survey on the straight and bent optical waveguides is discussed in Sec. 1.5. In Sec. 1.6 and Sec; 1.7, the scope and outline of the thesis are presented. In the last two sections, Sec. 1.8 and Sec. 1.9, we have listed the published work and concluded the chapter.

1.1 A BRIEF HISTORY OF MAXWELL'S EQUATIONS

During the second half of the 19th century, Maxwell unified the standard theories of electricity, magnetism, and light, which provided a complete description of electromagnetism phenomena and underpinned the theory of electromagnetic waves and their interaction with matter.

1.1.1 The Four Equations of Electromagnetic Theory

The fundamental theory of electromagnetic induction shows that the time-varying magnetic field produces the electric field and vice versa. Maxwell's equation refers to four relations that help us understand the properties and interrelations of electric and magnetic fields. All electromagnetic phenomena follow Maxwell's equations, which were constructed based on experimental observations.

$$\nabla \times \vec{\mathbb{E}} = -\frac{\partial \vec{\mathbb{B}}}{\partial t}, \quad \text{(Faraday's Law)}, \tag{1.1a}$$

$$\nabla \times \vec{\mathbb{H}} = \frac{\partial \vec{\mathbb{D}}}{\partial t} + \vec{\mathbb{J}}, \quad \text{(Ampere-Maxwell Law)}, \tag{1.1b}$$

$$\nabla \cdot \vec{\mathbb{D}} = \rho, \quad \text{(Gauss Law)}, \tag{1.1c}$$

$$\nabla \cdot \vec{\mathbb{B}} = 0, \quad \text{(Gauss Law for Magnetism)}. \tag{1.1d}$$

Here t is time, $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ represents the del operator, and $\vec{\mathbb{E}}$ and $\vec{\mathbb{H}}$ stands for the electric and magnetic field, $\vec{\mathbb{J}}$ is the current density, and ρ represent the free charge density. $\vec{\mathbb{D}} = \epsilon\vec{\mathbb{E}}$, and $\vec{\mathbb{B}} = \mu\vec{\mathbb{H}}$ are the electric displacement and the magnetic induction, ϵ and μ are the dielectric permittivity and magnetic permeability. The notation $\vec{\mathbb{E}} = \vec{\mathbb{E}}(x, y, z, t)$ is the vector function of the space and time variables, and the notation $\vec{E}$ is the vector function of the space variables. Similarly, for the other notations.

These four equations are divided into two pairs: Eq. (1.1a), (1.1b) and Eq. (1.1c), (1.1d). The first pair of equations describe how electric and magnetic fields are related. Eq.(1.1a) describes how a time-varying magnetic field will cause an electric field to curl around it, and Eq.(1.1b) describes how a magnetic field occurs around a time-varying electric field or electric current flowing through it. The second pair explains the electric force field surrounding the distribution of electric charge ρ, which shows electric field lines diverge from the area of the positive charge and converge onto an area of a negative charge, as shown in Fig. 1.1. Eq. (1.2d) shows that the magnetic field line curls to form closed loops, implying that the south pole accompanies every north pole of the magnet.

These four equations are solved only if $\vec{\mathbb{D}}, \vec{\mathbb{B}}, \vec{\mathbb{E}}$, and $\vec{\mathbb{H}}$ satisfy the constitutive relations i.e., if $\vec{\mathbb{D}}$ is related to $\vec{\mathbb{E}}$, $\vec{\mathbb{B}}$ is related to $\vec{\mathbb{H}}$, $\vec{\mathbb{J}}$ is related to $\vec{\mathbb{E}}$ in a specific way. That depends upon the properties of the medium, field strengths, e.g., for a linear, isotropic, homogeneous medium $\vec{\mathbb{D}} = \epsilon\vec{\mathbb{E}}$, $\vec{\mathbb{B}} = \mu\vec{\mathbb{H}}$, and $\vec{\mathbb{J}} = \sigma\vec{\mathbb{E}}$, where σ is the conductivity.

For a charge free dielectric $\sigma = 0$ and $\vec{\mathbb{J}} = 0$, Maxwell's equation becomes

$$\nabla \times \vec{\mathbb{E}} = -\mu\frac{\partial \vec{\mathbb{H}}}{\partial t}, \qquad \nabla \cdot (\epsilon\vec{\mathbb{E}}) = 0,$$

$$\nabla \times \vec{\mathbb{H}} = \epsilon\frac{\partial \vec{\mathbb{E}}}{\partial t}, \qquad \nabla \cdot \vec{\mathbb{H}} = 0.$$

For a non-conducting medium, i.e., not able to conduct electricity, such as plastic, glass,

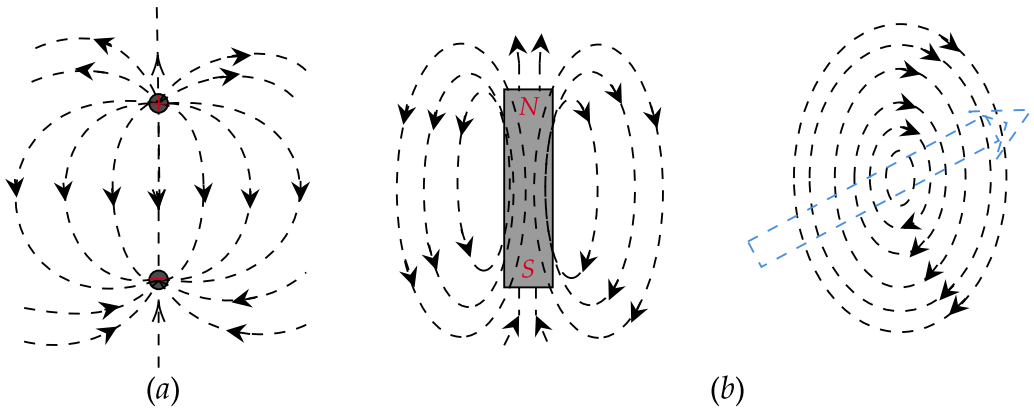


Figure 1.1. : (a) Behavior described by Eq. (1.1a), and Eq.. (1.1b). (b) Behaviour described by Eq. (1.1c), and Eq.. (1.1d). In the last figure, an arrow indicates the direction of the current.

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rubber, etc., a source-free medium, the frequency-varying Maxwell's equation becomes

$$\begin{aligned}
\nabla \times \vec{\mathbb{E}} &= -\imath\omega\mu\vec{\mathbb{H}}, \quad \text{(Faraday's law of electromagnetic induction)}, && (1.2a)\\
\nabla \times \vec{\mathbb{H}} &= \imath\omega\varepsilon\vec{\mathbb{E}}, \quad \text{(Ampere's law–current makes a magnetic field)}, && (1.2b)\\
\nabla \cdot \vec{\mathbb{E}} &= 0, \quad \text{(Gauss's law–charge makes an electric field)}, && (1.2c)\\
\nabla \cdot \vec{\mathbb{H}} &= 0, \quad \text{(The magnetic field is solenoidal)}, && (1.2d)
\end{aligned}$$

with the time-harmonic dependency as $\exp \imath\omega t$. Here, ω is the angular frequency. We will utilize Maxwell equations in this form in Sec. 1.3 and Sec. 1.4, which leads to the mathematical analysis of the bent waveguide model in Chap. 3 and Chap. 4.

1.1.2 Interface conditions for the electromagnetic fields

The differential forms of Maxwell's equations require that there is always an open neighborhood around a point where they are applied; otherwise, the vector fields $\mathbb{E}, \mathbb{D}, \mathbb{B}$, and $\mathbb{H}$ are not differentiable. The same situation occurs on the interface when two different mediums have different values for electrical permittivity and magnetic permeability.

Consider $\mathbb{E}_1$ and $\mathbb{E}_2$ are two electric field around the interface in medium 1 and medium 2. Then, the interface condition for electric field is $n_{12} \times (\mathbb{E}_2 - \mathbb{E}_1) = 0$. Here n_{12} is a normal vector from medium 1 to medium 2. Similarly, for the magnetic field $n_{12} \times (\mathbb{B}_2 - \mathbb{B}_1) = 0$. Therefore, the tangential components of electric field $\mathbb{E}$ and magnetic field $\mathbb{H}$ are continuous across the interface.

1.1.3 Plane-waves in Dielectric Medium

A plane wave in a dielectric medium is an electromagnetic wave with a constant frequency and wavelength and a flat wavefront, i.e., the wave propagates in a straight line with a uniform shape without spreading over time. These waves are useful in the theoretical construction to understand the behavior of the wave in free space or a uniform medium. In practical situations,

real electromagnetic waves are not plane waves but are still helpful for analyzing electromagnetic phenomena.

The relation derived in Eq. (1.3) and Eq. (1.4) states the relationship between the electric and magnetic fields in vector form. This explains that the electric and magnetic fields are perpendicular to each other, as well as to the direction of the propagation of the waves. For the plane-wave propagating in the direction of $\vec{k}$, the electric and magnetic fields can be of the form

$$\vec{\mathbb{E}} = \vec{E}_0 \exp \imath(\vec{k}\cdot\vec{r} + \omega t) = (E_{0x}, E_{0y}, E_{0z}) \exp \imath(k_x x + k_y y + k_z z + \omega t)$$
$$\vec{\mathbb{H}} = \vec{H}_0 \exp \imath(\vec{k}\cdot\vec{r} + \omega t) = (H_{0x}, H_{0y}, H_{0z}) \exp \imath(k_x x + k_y y + k_z z + \omega t)$$

Here $\vec{E}_0$ and $\vec{H}_0$ are space and time-independent vectors. From the Eq.(1.2c) and Eq.(1.2d), we get $\vec{k}\cdot\vec{\mathbb{E}} = 0$, $\vec{k}\cdot\vec{\mathbb{H}} = 0$ [Ghatak, 2017]. This shows that the electric and magnetic fields are perpendicular to the direction of the propagation, i.e., waves are transverse. From Eq. (1.2a) and Eq, (1.2b) by using the cross product formula and canceling the common terms on both sides, one gets

$$k_y E_{0z} - k_z E_{0y} = \omega\mu H_{0x},$$
$$k_z E_{0x} - k_x E_{0z} = \omega\mu H_{0y},$$
$$k_x E_{0y} - k_y E_{0x} = \omega\mu H_{0z}.$$

Therefore, for the magnetic field

$$\vec{\mathbb{H}} = \frac{\vec{\mathbb{E}} \times \vec{k}}{\omega\mu}. \tag{1.3}$$

Similarly, for the electric field,

$$\vec{\mathbb{E}} = \frac{\vec{k} \times \vec{\mathbb{H}}}{\omega\mu}. \tag{1.4}$$

If we suppose waves are propagating in the $z-$ direction and, at any instant, the electric field vector is along the $x-$axis, then the magnetic field vector will be along the $y-$axis.

Also, for the Eq. (1.3) and Eq. (1.4), for the magnitude of the fields, we get $k = \omega\sqrt{\epsilon\mu}$. The speed of propagation of electric and magnetic fields is given by $v = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}}$. Again, from Eq. (1.3) and Eq. (1.4),

$$\vec{H} = \frac{\vec{k} \times \vec{E}}{\omega\mu} = \frac{\vec{k}}{k} \times \vec{E}\frac{k}{\omega\mu} = \frac{1}{\eta}(\hat{k} \times E)$$

where $\eta = \frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\epsilon}}$, which is the intrinsic impedance of the medium. In a dielectric medium, $\mu = \mu_0$ and $n = \frac{\eta_0}{\eta}$, which is the refractive index of the dielectric medium.

According to the above relations, we got information about the behavior of the propagating electromagnetic fields in a dielectric medium. In the next section, we will discuss waveguides and their properties.

1.2 WAVEGUIDES AND THEIR APPLICATIONS: AN OVERVIEW

The optical waveguide is a physical structure or device designed to confine and guide the optical waves. The fundamental principle behind the optical waveguides is total internal reflection.

When light encounters the boundary between two materials with different refractive indices, and the angle of incidence is greater than the critical angle, the light is reflected into the higher refractive index material. This phenomenon allows light to be confined within a core material, preventing it from escaping and enabling it to propagate along the waveguide.

1.2.1 Classification of the waveguides

The waveguides are classified based on different parameters such as shape, material used, etc. From our point of interest, we will focus only on the three-layer dielectric waveguides and classify them further based on their shapes. A **'three-layer waveguide'** is a structure consisting of three regions: a substrate, a core or film (inner-most layer), and a cladding region. They are generally divided into symmetric and asymmetric waveguides. The **symmetric waveguides** are the ones that have the same refractive index of the substrate and cladding medium, whereas **asymmetric waveguides** consist of the different refractive index of all three regions. In the following subsection, we will further discuss the classification of the three-layer waveguides.

The **dielectric waveguides** are the waveguides made up of dielectric materials such as glass, polymers, etc. These waveguides have a higher refractive index in the inner region than in the outer regions, i.e., the core has a higher refractive index than the substrate and cladding medium.

Several types of waveguides are classified based on their structure, such as planner or slab waveguides, rectangular waveguides, circular waveguides, elliptical waveguides, and parallel waveguides. A **planner waveguide**, also called a "slab waveguide, is a type of dielectric waveguide that is flat and rectangular and consists of a thin layer of high refractive index material sandwiched between two layers of lower refractive index materials. This waveguide guides the waves in one dimension. A **circular waveguides** has a circular cross-section. Discussed waveguides are shown in Fig. 1.2.

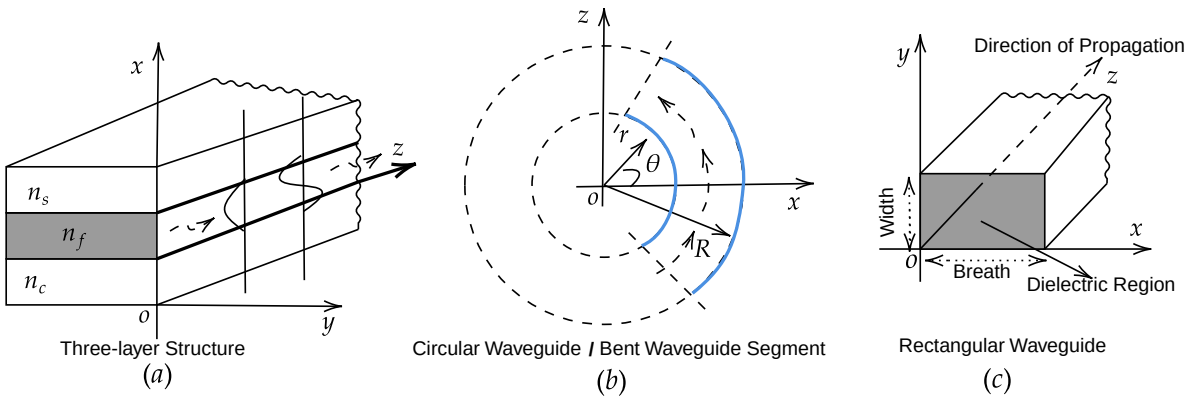


Figure 1.2. : (a) Three-layer planner or slab waveguide structure with n_s, n_f, and n_c are refractive index of the corresponding medium. Two modes are shown, representing the distribution of the field in the $x-$ direction. (b) bent waveguide with a radius R, enclosed by blue curves. In some applications, ring waveguides are considered, as shown in this figure, by ring constructed using dotted lines. (c) Rectangular waveguide.

In such waveguides, electromagnetic waves propagate in a bounded domain. In straight waveguides, there is no power loss due to radiation. However, in the case of bent waveguides, loss occurs due to the bending of the waveguides. The mathematical models created for a particular

waveguide describe the behavior of the waves, and the structure of the waveguide determines the types of modes that can propagate along it. The electromagnetic waves can be propagated in several modes within a waveguide, like transverse electric (TE), transverse magnetic (TM).

The TE wave has an electric field only in the transverse plane or perpendicular to the direction of propagation (i.e., longitudinal components $E_z = 0$ and $H_z \neq 0$). The TM waves have only the magnetic field in the transverse plane (i.e., in the direction of propagation, $H_z = 0$ and $E_z \neq 0$). These modes are obtained as the solution of Maxwell's equations.

The prominent waveguides are straight and bent. In the following two sections, we will discuss the wave propagation behavior in both waveguides and construct a mathematical model for each. These models will be helpful for mathematical analysis in Chap. 3, and Chap. 4 of this thesis.

1.2.2 Characteristics of waves propagation along uniform waveguide structure

Consider an electromagnetic wave propagating in a waveguide in $z-$direction (only one direction) with propagation constant $\gamma = \beta + \imath\alpha$. For time-harmonic dependence with the angular frequency ω and dependence on z and t, all field components are given as: in Cartesian coordinate,

$$\vec{\mathbb{E}}(x, y, z, t) = \vec{E}(x, y) \exp(\imath\omega t - \gamma z).$$

Similarly,

$$\vec{\mathbb{H}}(x, y, z, t) = \vec{H}(x, y) \exp(\imath\omega t - \gamma z).$$

Consider a dielectric straight waveguide having a 1-D cross-section and lying along the z-axis as shown in Fig. (1.3).

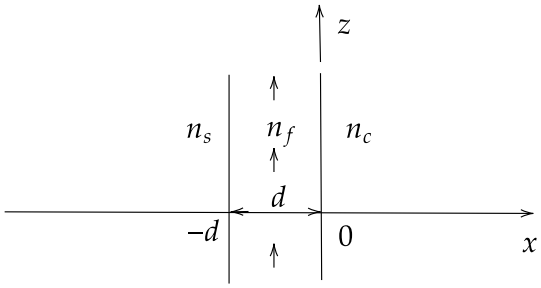


Figure 1.3. : Straight waveguide setup with (n_s, n_f, n_c) substrate, film, and cladding refractive index profile, d is the core width, and the waves are propagating in positive $z-$direction.

From Maxwell's equation, for the electric and magnetic field intensities in the charge-free dielectric region and by the solution representation above, we get

$$\begin{aligned} \nabla_{xy}^2 \vec{E} + (k^2 + \gamma^2)\vec{E} = 0, \\ \nabla_{xy}^2 \vec{H} + (k^2 + \gamma^2)\vec{H} = 0, \end{aligned} \tag{1.5}$$

where $\nabla^2 = \nabla_{xy}^2 + \nabla_z^2$. The above equations are second-order partial differential equations for each

component of $\vec{E}$ and $\vec{H}$.

$$\frac{\partial^2 E_x}{\partial^2 x} + \frac{\partial^2 E_x}{\partial^2 y} + (k^2 + \gamma^2)E_x = 0,$$
$$\frac{\partial^2 E_y}{\partial^2 x} + \frac{\partial^2 E_y}{\partial^2 y} + (k^2 + \gamma^2)E_y = 0,$$
$$\frac{\partial^2 E_z}{\partial^2 x} + \frac{\partial^2 E_z}{\partial^2 y} + (k^2 + \gamma^2)E_z = 0.$$

Similarly, we also get three partial differential equations for the magnetic field. The exact solution of these equations depends upon the cross-section geometry and the boundary conditions of components at material interfaces.

Again, from Maxwell's equation and the solution representations

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -\imath\omega\mu H_x, \quad (1.6a) \qquad \frac{\partial H_z}{\partial y} + \gamma H_y = \imath\omega\epsilon E_x, \quad (1.7a)$$
$$-\gamma E_x - \frac{\partial E_z}{\partial x} = -\imath\omega\mu H_y, \quad (1.6b) \qquad -\gamma H_x - \frac{\partial H_z}{\partial x} = \imath\omega\epsilon E_y, \quad (1.7b)$$
$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\imath\omega\mu H_z, \quad (1.6c) \qquad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \imath\omega\epsilon E_z. \quad (1.7c)$$

These equations express the transverse field components H_x, H_y and E_x, E_y in terms of the two longitudinal components E_z and H_z. In Eq. (1.6a), put the value of E_y to express H_x in terms of E_z and H_z and similarly for the other field components. We get

$$H_x = -\frac{1}{\gamma^2 + k^2}\left(-\imath\epsilon\omega\frac{\partial E_z}{\partial y} + \gamma\frac{\partial H_z}{\partial x}\right), \qquad E_x = -\frac{1}{\gamma^2 + k^2}\left(\imath\mu\omega\frac{\partial H_z}{\partial y} + \gamma\frac{\partial E_z}{\partial x}\right),$$
$$H_y = -\frac{1}{\gamma^2 + k^2}\left(\imath\epsilon\omega\frac{\partial E_z}{\partial x} + \gamma\frac{\partial H_z}{\partial y}\right), \qquad E_y = \frac{1}{\gamma^2 + k^2}\left(-\imath\mu\omega\frac{\partial H_z}{\partial x} + \gamma\frac{\partial E_z}{\partial y}\right). \quad (1.8)$$

The wave behavior in a waveguide can be analyzed by Eq.(1.5) for the longitudinal components E_z and H_z and then by Eq.(1.8) to determine the other components.

1.2.3 Classification of propagating waves in uniform waveguide

The propagation of the waves in the waveguide is classified according to the existence of the longitudinal components E_z and H_z, namely (1) TEM waves (2) TM waves (3) TE waves. Since the TEM waves do not exist in the 1-D waveguides, we only discussed the other two wave types for this waveguide.

Transverse Magnetic waves (TM): For TM waves, the longitudinal component of the magnetic field H_z is zero in the direction of propagation, which can be analyzed by the Eq. (1.5) for the E_z component with boundary conditions and Eq. (1.8). Therefore, by Eq. (1.5) for the longitudinal component of the electric field

$$\nabla_{xy}^2 E_z + (k^2 + \gamma^2)E_z = 0.$$

For TM, $H_z = 0$,

$$H_x = \frac{\imath\epsilon\omega}{\gamma^2 + k^2}\frac{\partial E_z}{\partial y} \quad (1.9a) \qquad E_x = -\frac{\gamma}{\gamma^2 + k^2}\frac{\partial E_z}{\partial x} \quad (1.10a)$$
$$H_y = -\frac{\imath\epsilon\omega}{\gamma^2 + k^2}\frac{\partial E_z}{\partial x} \quad (1.9b) \qquad E_y = -\frac{\gamma}{\gamma^2 + k^2}\gamma\frac{\partial E_z}{\partial y} \quad (1.10b)$$

From the above equations for the transverse components of the electric field

$$a_x E_x + a_y E_y = -\frac{\gamma}{\gamma^2 + k^2}\left(a_x \frac{\partial E_z}{\partial x} + a_y \frac{\partial E_z}{\partial y}\right),\ (E_T)_{TM} = -\frac{\gamma}{\gamma^2 + k^2}\nabla_T E_z.$$

Consider $h^2 = \gamma^2 + k^2$. We get $\gamma = \sqrt{h^2 - \omega^2\mu\epsilon}$. From this, $\gamma = 0$, if $h^2 = \omega_c^2\mu\epsilon$, or $f_c = \frac{h}{2\pi\sqrt{\mu\epsilon}}$. The frequency f_c, where $\gamma = 0$ is known as the cutoff frequency. For a particular mode in a waveguide, the value of f_c depends on the eigenvalue of the mode. From these expressions, one can write as

$$\gamma = h\sqrt{1 - \left(\frac{f}{f_c}\right)^2}.$$

The above expression helps us to divide the two distinct ranges of γ depending on the sign of $1 - \left(\frac{f}{f_c}\right)^2$, which leads to the discussion on propagation constant, wavelength, and phase velocity [Cheng, 1989].

Transverse electric waves (TE): For TE waves, the longitudinal component of the electric field E_z is zero in the direction of propagation, which can be analyzed by the Eq. (1.5) for the H_z component with boundary conditions and Eq. (1.8). Therefore, by Eq. (1.5) for the longitudinal component of the electric field

$$\nabla_{xy}^2 H_z + (k^2 + \gamma^2)H_z = 0.$$

For TE, $E_z = 0$,

$$H_x = -\frac{1}{\gamma^2 + k^2}\left(\gamma \frac{\partial H_z}{\partial x}\right), \qquad E_x = -\frac{1}{\gamma^2 + k^2}\left(\imath\mu\omega \frac{\partial H_z}{\partial y}\right),$$
$$H_y = -\frac{1}{\gamma^2 + k^2}\left(\gamma \frac{\partial H_z}{\partial y}\right), \qquad E_y = \frac{1}{\gamma^2 + k^2}\left(\imath\mu\omega \frac{\partial H_z}{\partial x}\right).$$

From the above equations for the transverse components of the magnetic field

$$a_x H_x + a_y H_y = -\frac{\gamma}{\gamma^2 + k^2}\left(a_x \frac{\partial H_z}{\partial x} + a_y \frac{\partial H_z}{\partial y}\right),\ (H_T)_{TE} = -\frac{\gamma}{\gamma^2 + k^2}\nabla_T H_z.$$

Similarly, as above, one can also find the cutoff frequency, propagation constant, wavelength, and phase velocity, in this case, [Cheng, 1989].

The above discussion helps us understand how the wave propagates in the waveguide and the necessary conditions for its propagation. We are omitting the case of TEM because of our interest in dielectric waveguides. In the next section, we will discuss the models of straight and bent waveguides.

1.3 STRAIGHT WAVEGUIDES

A straight optical waveguide is designed to guide and transmit light along a straight path with minimal loss. It consists of a dielectric material with a higher refractive index than its surrounding medium, which allows total internal reflection to occur at the interface between the core and cladding layers.

The core of the waveguide is typically made of a dielectric material with a high refractive index. At the same time, the cladding layer has a lower refractive index to confine the light within the core. This refractive index contrast ensures that light is effectively trapped and guided along the length of the waveguide. In this section, we will discuss the three-layer 1-D straight waveguide, in which the refractive index profile varies only in one direction; here in x-direction.

1.3.1 Model of straight waveguides

Assume that the electromagnetic fields and the material properties are not varying in the $y-$ direction and waves are propagating in the $z-$ direction shown in Fig. 1.3. Consider n_s, n_f, and n_c as the refractive index for the substrate, film, and cladding medium. The refractive index profile $n(x)$ is

$$n(x) = \begin{cases} n_s & -\infty < x < -d, \\ n_f & -d \leq x \leq 0, \\ n_c & 0 < x < \infty. \end{cases} \tag{1.11}$$

Assume that the electromagnetic fields with the time-harmonic exponential dependence $\exp(\imath\omega t)$ and the wave is propagating along the $z-$direction with exponential dependence as $\exp(-\imath\beta_s z)$ is represented by

$$\begin{aligned} \vec{\mathbb{E}}(x,y,z,t) &= \vec{\mathbb{E}}(x,z,t) && (1.12)\\ &= \vec{E}(x,z)\exp(\imath\omega t), \\ &= \tilde{E}(x)\exp\imath(\omega t - \beta_s z) && (1.13)\\ &= (E_x, E_y, E_z)(x)\exp\imath(\omega t - \beta_s z), && (1.14)\\ \vec{\mathbb{H}}(x,y,z,t) &= \vec{\mathbb{H}}(x,z,t) && (1.15)\\ &= \vec{H}(x,z)\exp(\imath\omega t), \\ &= \tilde{H}(x)\exp\imath(\omega t - \beta_s z) && (1.16)\\ &= (H_x, H_y, H_z)(x)\exp\imath(\omega t - \beta_s z), && (1.17) \end{aligned}$$

where E_x, E_y, E_z and H_x, H_y, H_z are the electric and magnetic field components in x, y and z directions, and β_s is the propagation constant for the straight waveguide. Using vector cross product formula and Eq. (1.14) and Eq. (1.17), one gets

$$\nabla \times \vec{\mathbb{E}} = \left(\imath\beta_s E_y, -\frac{\partial E_z}{\partial x} - \imath\beta_s E_x, \frac{\partial E_y}{\partial x}\right)\exp\imath(\omega t - \beta_s z).$$

Similarly,

$$\nabla \times \vec{\mathbb{H}} = \left(\imath\beta_s H_y, -\frac{\partial H_z}{\partial x} - \imath\beta_s H_x, \frac{\partial H_y}{\partial x}\right)\exp\imath(\omega t - \beta_s z).$$

From Eq. (1.2a) and Eq. (1.2b),

$$\beta_s E_y = -\omega\mu H_x, \tag{1.18a}$$

$$\frac{dE_z}{dx} + \imath\beta_s E_x = \imath\omega\mu H_y, \tag{1.18b}$$

$$\frac{dE_y}{dx} = -\imath\omega\mu H_z, \tag{1.18c}$$

and

$$\beta_s H_y = \omega\varepsilon E_x, \tag{1.19a}$$

$$\frac{dH_z}{dx} + \imath\beta_s H_x = -\imath\omega\varepsilon E_y, \tag{1.19b}$$

$$\frac{dH_y}{dx} = \imath\omega\varepsilon E_z. \tag{1.19c}$$

Therefore, the TE mode profiles are given as

$$\begin{aligned} &\tilde{E}_x(x,z) = 0, && \tilde{H}_x(x,z) = H_x(x)e^{-\imath\beta_s z} = -\frac{\beta_s}{\mu\omega}E_y(x)e^{-\imath\beta_s z}, \\ &\tilde{E}_y(x,z) = E_y(x)e^{-\imath\beta_s z}, && \tilde{H}_y(x,z) = 0, \\ &\tilde{E}_z(x,z) = 0, && \tilde{H}_z(x,z) = H_z(x)e^{-\imath\beta_s z} = \frac{\imath}{\mu\omega}\frac{dE_y(x)}{dx}e^{-\imath\beta_s z}, \end{aligned} \tag{1.20}$$

and we also get one more equation from Eq. (1.19b)

$$\frac{d^2E_y}{dx^2} + \omega\mu\varepsilon E_y = \beta_s^2 E_y, \tag{1.21}$$

which is a $1-D$ eigenvalue problem for the TE modes of the straight waveguide in terms of the electric field component in $y-$direction and $\mu = \mu_0\mu_r$, $\varepsilon = \varepsilon_0\varepsilon_r$, with $\varepsilon_r = n^2(x)$ as in Eq. (1.11). Similarly, TM mode profiles are given as

$$\begin{aligned} &\tilde{E}_x(x,z) = E_x(x)e^{-\imath\beta_s z} = \frac{\beta_s}{\omega\varepsilon}H_y(x)e^{-\imath\beta_s z}, && \tilde{H}_x(x,z) = 0, \\ &\tilde{E}_y(x,z) = 0, && \tilde{H}_y(x,z) = H_y(x)e^{-\imath\beta_s z}, \\ &\tilde{E}_z(x,z) = E_z(x)e^{-\imath\beta_s z} = \frac{1}{\imath\omega\varepsilon}\frac{\partial H_y}{\partial x}e^{-\imath\beta_s z}, && \tilde{H}_z(x,z) = 0. \end{aligned} \tag{1.22}$$

and similar as above, we also get one more equation from Eq. (1.18b)

$$\varepsilon\frac{d}{dx}\left(\frac{1}{\varepsilon}\frac{dH_y}{dx}\right) + \omega^2\mu\varepsilon H_y = \beta_s^2 H_y, \tag{1.23}$$

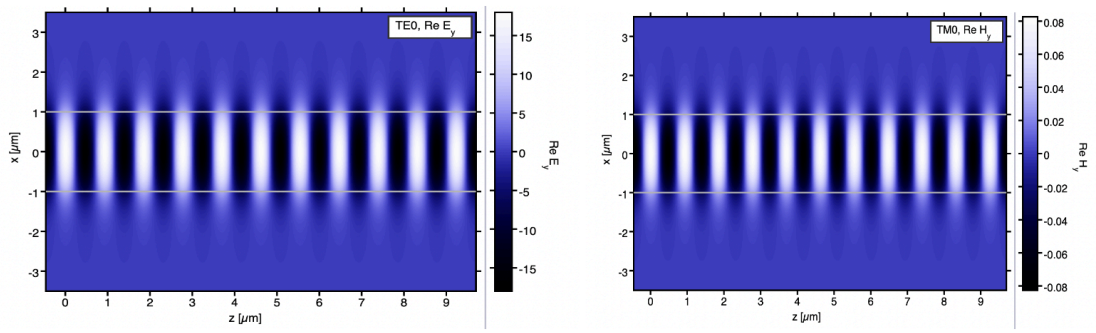
which is a $1-D$ eigenvalue problem for the TM modes of the straight waveguide in terms of the magnetic field component in the $y-$ direction. The eigenvalue problems (1.21) and (1.23) for the TE and TM modes are the same, differ only with the first term expression. The boundary conditions for both the eigenvalue problem is

$$|\psi(x)| \to 0 \text{ as } x \to \pm\infty,$$

and ψ is an outgoing wave. The field profiles, as shown in Fig. (1.5) for each component of the electromagnetic field, are expressed in Eq. (1.20) and Eq.(1.22) for the specific values of the parameters. The mode profile for TE_0 and TM_0 modes propagating in a straight waveguide are shown in Fig. 1.4.

1.4 BENT WAVEGUIDES

A bent waveguide is a type of waveguide with a curved shape instead of a straight (see Fig. (1.6)). Bent waveguides are commonly used in many applications, such as in optical communications,



(a) (b)

Figure 1.4. : Thickness $d = 2\mu m$, $(n_s, n_f, n_c) = (1.6, 1.7, 1.55)$, Vacuum wavelength $= 1.55\mu m$. Electromagnetic field profile (a) Fundamental transverse electric mode (TE_0) profile of straight waveguide (b) Fundamental transverse magnetic mode (TM_0) profile of straight waveguide. For figure see web-page [Hammer, 2023]

where they allow light to be directed around corners and bends without significant loss or distortion. The design of bent waveguides is an essential topic in waveguide engineering, and many different types of bending techniques can be used to create them, including gradual bends, curved waveguide sections, and angled bends. The choice of bending technique depends on the specific application and the desired performance characteristics of the waveguide.

1.4.1 Bent waveguide model

Consider a 1-D bent waveguide with the piece-wise constant radial refractive index profile as shown in Fig. 1.6. Assume that the mode propagates in the positive z-direction, and the material properties do not vary in the y-direction. The original problem in the xz plane is modeled in the polar coordinates (r, θ).

Let the form of the propagating electric field $(\vec{\mathbb{E}})$ and the magnetic field $(\vec{\mathbb{H}})$ be as

$$\vec{\mathbb{E}} = \vec{\mathbb{E}}(r, y, \theta, t) = (\tilde{E}_r(r), \tilde{E}_y(r), \tilde{E}_\theta(r))e^{\imath(\omega t - \gamma R\theta)}, \tag{1.24a}$$

$$\vec{\mathbb{H}} = \vec{\mathbb{H}}(r, y, \theta, t) = (\tilde{H}_r(r), \tilde{H}_y(r), \tilde{H}_\theta(r))e^{\imath(\omega t - \gamma R\theta)}. \tag{1.24b}$$

Here $\tilde{E}_r(r)$, $\tilde{E}_y(r)$, and $\tilde{E}_\theta(r)$ are mode profile components of the electric field in r, y and θ directions respectively. A similar convention is also applied to the magnetic field. ω is the angular frequency corresponding to given vacuum wavelength λ. γ is a bent mode propagation constant, which is a prior unknown, and R is the bend radius.

Semi-analytical studies of the bent waveguide problem showed that the bent modes are lossy [Hiremath *et al.*, 2005], which was modeled as the non-real-valued mode propagation constants. Without loss of generality, assume $\gamma = \pm\beta \pm \imath\alpha$, where α may or may not be zero.

In the above setting, we assume that the modes propagate in the anti-clockwise direction. For such modes, the phase constant $=\mathrm{Re}(\gamma) > 0$. For the physically stable modes (i.e., modes with finite energy), electromagnetic fields can not grow as $r \to \infty$ (i.e., in the cladding region). Therefore, for the above functional form for the electric and magnetic fields, one concludes that if the attenuation constant $= \mathrm{Im}(\gamma)$ is non-zero, it must be negative. At the present moment, we are not ruling out the possibility that α can be zero. Thus, we use the following form of the bent mode

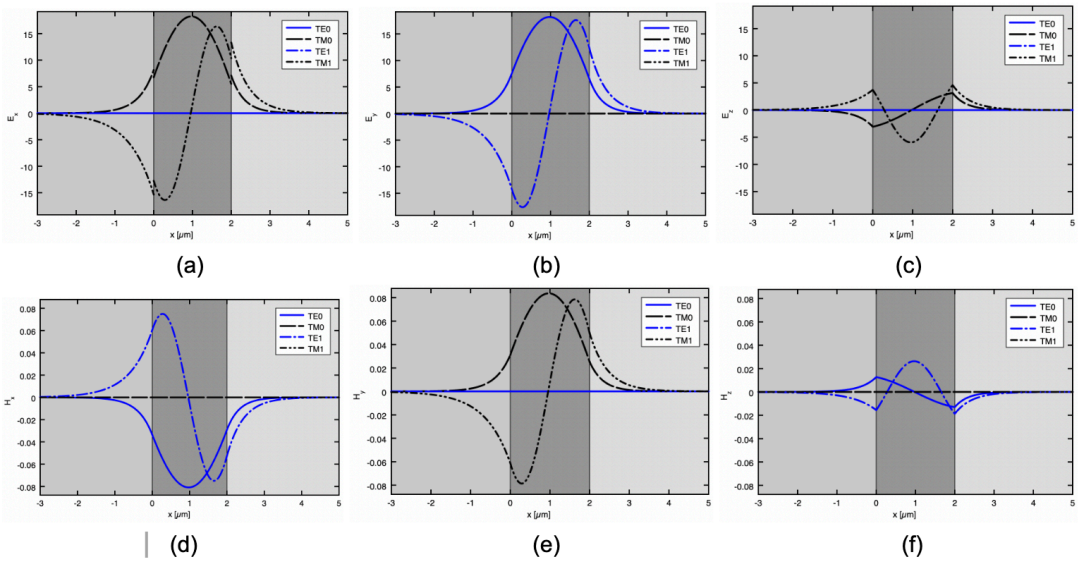


Figure 1.5. : Thickness $d = 2\mu m, (n_s, n_f, n_c) = (1.6, 1.7, 1.55)$, vacuum wavelength $= 1.55\mu m$. Electromagnetic field components for the first two fundamental modes (a) E_x (b) E_y (c) E_z (d) H_x (e) H_y (f) H_z components respectively. For plot see web-page [Hammer, 2023]

propagation constant:

$$\gamma = \beta - \imath\alpha \qquad \beta > 0,\ \alpha \geq 0, \tag{1.25}$$

Now, we will plug the field ansatz Eq. (1.24) in Maxwell's equations Eq. (1.2) to obtain the mathematical model for the wave propagation in the bent waveguide. Using the vector product in cylindrical coordinates

$$\vec{\nabla} \times \vec{E} = \frac{1}{r} \begin{vmatrix} \hat{r} & \hat{y} & r\hat{\theta} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial y} & \frac{\partial}{\partial \theta} \\ \tilde{E}_r & \tilde{E}_y & r\tilde{E}_\theta \end{vmatrix},$$

based on the model setup, and Eq. (1.24a) and Eq. (1.24b), one gets

$$\nabla \times \vec{E} \quad = \quad \left(-\frac{\imath\gamma R}{r}\tilde{E}_y, -\frac{1}{r}\frac{\partial r\tilde{E}_\theta}{\partial r} - \frac{\imath\gamma R}{r}\tilde{E}_r, \frac{\partial \tilde{E}_y}{\partial r} \right) \exp \imath(\omega t - \gamma R\theta).$$

Similarly,

$$\nabla \times \vec{H} = \left(-\frac{\imath\gamma R}{r}\tilde{H}_y, -\frac{1}{r}\frac{\partial r\tilde{H}_\theta}{\partial r} - \frac{\imath\gamma R}{r}\tilde{H}_r, \frac{\partial \tilde{H}_y}{\partial r} \right) \exp \imath(\omega t - \gamma R\theta).$$

From Eq. (1.2a) and Eq. (1.2b),

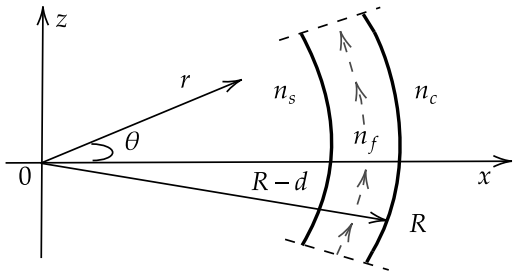


Figure 1.6. : 1-D bent waveguide set-up: A film of refractive index n_f and width d is sandwiched between the cladding of a refractive index of n_c and the substrate of a refractive index n_s. The radius of the bent waveguide R is measured as the distance of the outer interface from the origin, waves assumed to be propagating anti-clockwise in $z-$ direction.

$$\frac{\gamma R}{r}\tilde{E}_y = -\omega\mu\tilde{H}_r, \tag{1.26a}$$

$$\frac{1}{r}\frac{\partial r\tilde{E}_\theta}{\partial r} + \frac{\imath\gamma R}{r}\tilde{E}_r = \imath\omega\mu\tilde{H}_y, \tag{1.26b}$$

$$\frac{\partial \tilde{E}_y}{\partial r} = -\imath\omega\mu\tilde{H}_\theta, \tag{1.26c}$$

$$\frac{\gamma R}{r}\tilde{H}_y = \omega\varepsilon\tilde{E}_r, \tag{1.27a}$$

$$\frac{1}{r}\frac{\partial r\tilde{H}_\theta}{\partial r} + \frac{1}{r}\imath\gamma R\tilde{H}_r = -\imath\omega\varepsilon\tilde{E}_y, \tag{1.27b}$$

$$\frac{\partial \tilde{H}_y}{\partial r} = \imath\omega\varepsilon\tilde{E}_\theta. \tag{1.27c}$$

For the transverse electric (TE) waves, the only non-zero components of an electromagnetic field are $\tilde{E}_y, \tilde{H}_r$, and $\tilde{H}_\theta$. We get from the remaining equations, i.e., Eq. (1.26a), Eq. (1.26c), and Eq. (1.27b)

$$\frac{d^2\tilde{E}_y}{dr^2} + \frac{1}{r}\frac{d\tilde{E}_y}{dr} + \omega\mu\varepsilon\tilde{E}_y = \frac{\gamma^2 R^2}{r^2}\tilde{E}_y, \tag{1.28}$$

which is a $1-D$ eigenvalue problem for the bent waveguide in terms of the electric field component in $y-$direction and $\mu = \mu_0\mu_r$, $\varepsilon = \varepsilon_0\varepsilon_r$, with $\varepsilon_r = n^2(r)$ such as

$$n(r) = \begin{cases} n_s & 0 < r < R-d, \\ n_f & R-d \le r \le 0, \\ n_c & 0 < r < \infty. \end{cases} \tag{1.29}$$

Similarly, for the transverse magnetic (TM) waves, the only zero components of an electromagnetic field are $\tilde{H}_y, \tilde{E}_r$, and $\tilde{E}_\theta$. From the remaining equations (i.e., Eq. (1.26b), Eq. (1.27a), and Eq. (1.27c)), one gets one will get 1-D eigenvalue problem for the magnetic field component in $y-$direction

$$\varepsilon r\left[\frac{\partial}{\partial r}\left(\frac{r}{\varepsilon}\frac{d\tilde{H}_y}{dr}\right)\right] + \mu\omega^2 r^2\varepsilon\tilde{H}_y = \gamma^2 R^2\tilde{H}_y. \tag{1.30}$$

The above equation will change into a simpler form in the case of the constant step-index profile, as in Eq. (1.28). The boundary conditions for the guided modes are $|\psi(r)| \to 0$ as $r \to 0, |\psi(r)| \to 0$ as $r \to \infty$ and ψ is an outgoing wave. The field for each component of the electromagnetic field is shown in Fig. 1.8 with the same parameter (which are common) values as taken for the case of the straight waveguide. The snapshots of the propagating fundamental bend modes according to Eq. (1.24) as shown in Fig. 1.7. The color scales correspond to the real, physical field levels E_y and H_y.

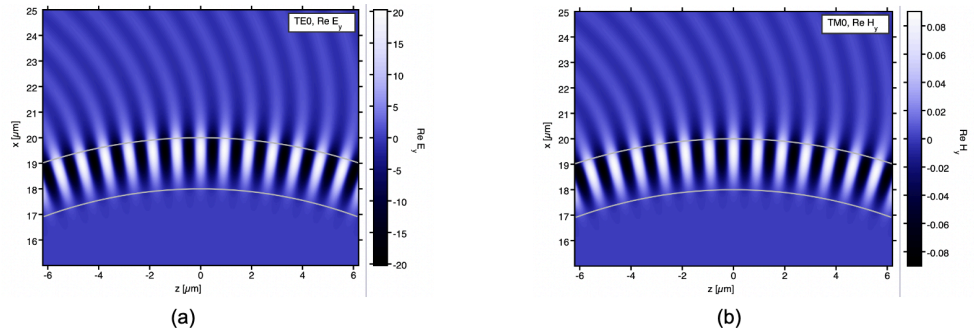


Figure 1.7. : Bent waveguide (a) TE_0 mode profile (b) TM_0 mode profile. Thickness $d = 2\mu m$, $R = 20$, $(n_s, n_f, n_c) = (1.6, 1.7, 1.55)$, Vacuum wavelength $= 1.55\mu m$. For plot see web-page [Hammer, 2023].

1.5 LITERATURE REVIEW: WAVEGUIDES

The study of optical wave propagation in waveguides is a versatile field for researchers, exploring experimental, numerical, and semi-analytic analytic studies. Our primary focus is to analyze the optical bent waveguide model analytically, which was first experimentally studied by Marcatili [Marcatili, 1969]. As the discussion in this thesis is based on the analytic study, we divided the literature survey into two parts, one in this section based on waveguides, and the second based on the non-self-adjoint problems, discussed in Sec. 2.4 in the next chapter. Several researchers investigated these waveguides using various techniques. In this section, we will give an overview of the studies on optical waveguides, starting from the straight waveguides.

Straight Waveguides :

Various forms of straight waveguides and optical fibers are studied quite well in terms of their mathematical models, numerical simulation schemes, and fabrications [Okamoto, 2006]. Modes of three-layer one-dimensional straight waveguides are studied using ray optics and wave optics-based approach [Hunsperger, 1995]. Using it, one can compute mode propagation constants and mode profiles and check their orthogonality.

The wave propagation in a 2-D optical straight waveguide model was studied in [Santosa and Magnanini, 2001]. The study was based on transformation theory, through which the solution for the time-harmonic wave equation in a waveguide is represented. The decay rate is derived for the non-guided part of the solution. The solution is also studied using Green's functions approach. In [Joly and Poirier, 1995], the mathematical analysis of electromagnetic open waveguides was done, where the behavior of the guided modes in $3-D$ electromagnetic media, which are invariant under translation in only one space direction, and the properties are discussed. In this work, the self-adjointness of the governing operator is proved by coercivity. The condition used to analyze the guided modes is the divergence-free condition in the functional framework (i.e., $\mathrm{div}_{\beta_s}(\varepsilon E) = 0$). Various results on guided modes are discussed, such as what happens at low and high frequencies, and characterize the eigenvalue through the Max-Min principle.

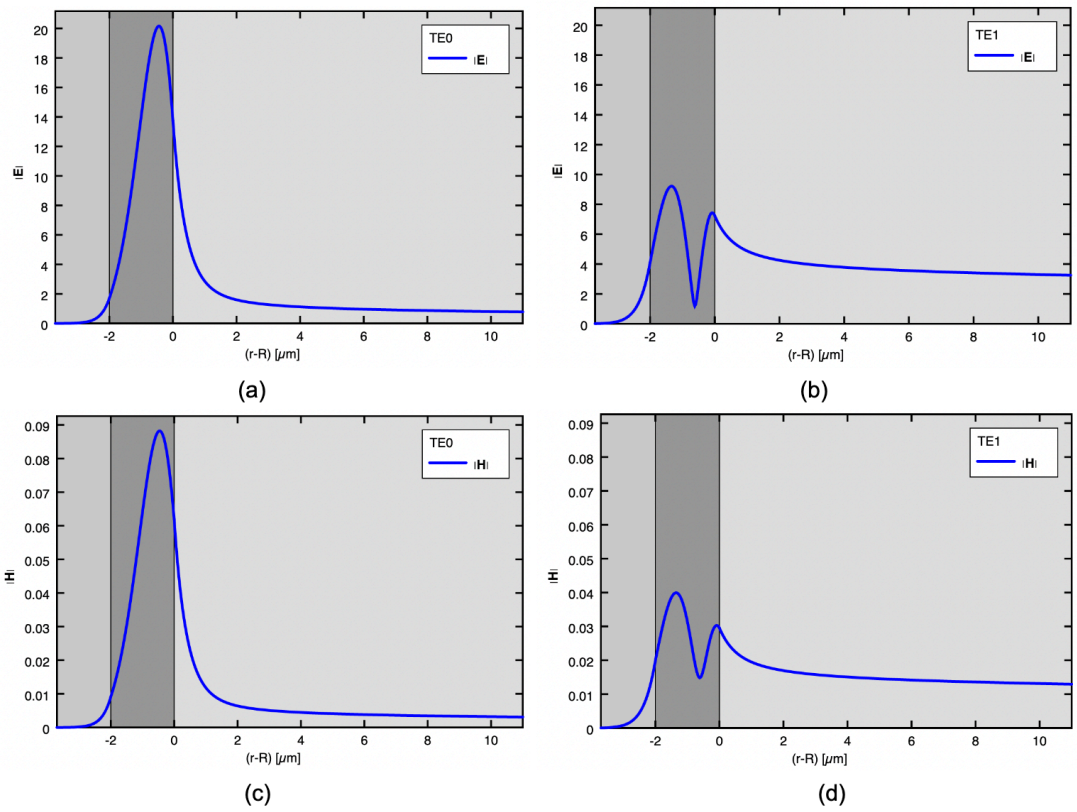


Figure 1.8. : For TE_0 and TE_1 mode, (a) and (b) represents the modulus of electric field, and (c) and (d) represents the modulus of magnetic field. Thickness $d = 2\mu m$, $R = 20$, $(n_s, n_f, n_c) = (1.6, 1.7, 1.55)$, Vacuum wavelength $= 1.55\mu m$. For plot see web-page [Hammer, 2023].

Bent Waveguides:

Another essential form of optical waveguides is bent optical waveguides. The bent waveguides were first introduced by Marcatili [Marcatili, 1969]. The lossy nature of the guided modes of these waveguides has always been the topic of special interest [Marcatili, 1969; Marcuse, 1971; Lewin, 1974]. Marcatili showed that power loss in the bent slab waveguide is due to radiation and the effects of bending of a dielectric slab in cylindrical coordinates. The problem was analyzed by solving the eigenvalue problem for the bent waveguide by an approximate theory [Marcuse, 1971]. Including the bending loss formula, expressions for the guided-mode loss in a circularly curved section of slab waveguide have been obtained in cylindrical coordinates from the wave equation.

In [Marcuse, 1971], a bent slab waveguide was described with a cylindrical coordinate system. An analysis based on perturbation methods to find the radiation conditions for the radiation from curved radiating structures was presented in [Lewin, 1974]. This analysis is the same as the method applied in [Marcatili, 1969], to obtain an approximation of the leakage from a bent light guide of a rectangular cross-section. The radiation conditions for a toroidal structure were also discussed [Marcatili, 1969].

A different approach proposed by Taylor based on the power loss at a directional change in dielectric waveguides [Taylor, 1974] where the gradual curves are treated by a coupled-mode approach based on the assumption that the lengths of the straight segments approach zero as the

number of corners becomes infinite. The guided-mode power loss at a directional change in dielectric waveguides can depend strongly on whether the angular displacement occurs as a gradual curve or a sequence of corners. He also treated the bending problem using a computer-based analysis [Taylor, 1974]. Taylor further extended his work published in [Taylor, 1974] and explained in detail about the bend loss by adding the loss at the corners of the waveguides. The loss at a corner bend in a single-mode fiber was calculated by evaluating a spatial integration of a function representing the perturbation multiplied by the square of the electric field distribution for the fundamental mode. He also showed the effect of coherent coupling between radiation and guided modes. The results were also numerically verified by applying them to the symmetric slab waveguides.

Using a conformal transformation, the bent waveguide eigenvalue problem is transformed into a study of the eigenvalue problem of the equivalent straight waveguide [Heiblum and Harris, 1975]. In this paper, the different kinds of structures of the bent waveguide are discussed with the help of the transformation. Kim and Gopinath investigated bent waveguides using the finite difference method [Kim and Gopinath, 1996]. Treating the bent waveguide as a perturbation of a straight waveguide, Melloni expressed bent waveguide modes in the straight waveguide modes [Melloni *et al.*, 2001].

In another approach, these bent waveguides were modeled semi-analytically. The mode propagation constants were found by solving the dispersion equation; the non-real-valued propagation constants were obtained, and the power orthogonality of the modes was studied [Hiremath *et al.*, 2005]. Continuing that work, we investigated this problem as an eigenvalue problem defined on an unbounded domain [Kumar and Hiremath, 2022].

1.6 SCOPE OF THE THESIS: MOTIVATION AND OBJECTIVES

Optical waveguides have become an essential component of all-optical signal processors. In the future, optical waveguides are expected to play an even more significant role in various fields.

Coming to the simplest one waveguides: straight waveguides. These waveguides have been studied in depth in the last few decades. Like straight waveguides, bent waveguides are also explored experimentally, numerically, and semi-analytically. But, a thorough mathematical analysis of bent waveguide eigenvalue problems has not been done. One of the challenges behind it is the non-self-adjointness of the operator involved, which is investigated in this thesis.

The study presented in this thesis is to analyze the bent waveguide model, where we will answer the following questions on constructing the eigenvalue problem corresponding to the model:

- What is the behavior of the adjointness of the operator corresponding to the governing eigenvalue problem?
- What is the nature of the eigenvalues of the eigenvalue problem?
- Mathematical evidence on the behavior of the bent waveguide problem with the bent radius R and its eigenvalues, i.e., asymptotic analysis of the bent waveguide model w.r.t. R.
- Boundedness of the eigenvalues of the problem and orthogonality of its eigenfunctions corresponding to distinct eigenvalues.
- Is there any explicit relation between phase propagation constant β, and attenuation constant α ?

Further, some questions still need to be explored, such as: how can one use the pseudospectra

and pseudo-functions of the problem to gain insight into the behavior of the model as a whole? How can pseudospectra of a non-self-adjoint problem help us to design control strategies for the model? The turning point of the bent waveguides problem, where the modes change their behavior, still needs to be explored. Can the analysis of the pseudo-spectra of non-self-adjoint problems provide insights into the design of new waveguides or other optical devices, and how can these insights be translated into practical applications?

In this thesis, we have restricted ourselves to the analysis of the operator. In the future, we will explore the questions that arise based on the pseudo-spectra analysis and try to answer them.

1.7 OUTLINE OF THE THESIS

The primary objective of this thesis is to conduct an analytical study of the optical bent waveguide model. Why do an analytic study? Because an analytic research can give further insight into the model compared to an experimental, numerical, or semi-analytic study. This study aims to answer questions such as adjointness of the governing model problem, eigenvalues behavior, relationship between phase propagation and attenuation constant (i.e., the real and imaginary part of the propagation constant), orthogonality of the eigenfunctions (=modes) corresponding to distinct eigenvalues, pseudospectra of the problem and insight into this for the model, turning points of the model, and so on. One can answer these questions through an analytical analysis of the model, which other methods can not obtain. Some questions still need to be answered in this work, leading to further analysis based on spectral analysis of the non-self-adjoint operators.

A reader of this thesis work will encounter the mathematical interpretation and physical interpretation of the concept of the straight and bent optical waveguides model. Investigating the mathematical aspects of the model and its relationship to the underlying physics provides a unique perspective on the subject matter. To facilitate a clear and organized presentation of our findings, this thesis is structured as follows:

- **Chapter 1:** This chapter offers an understanding of the key principles and applications of waveguides that are basic for understanding the work discussed in this thesis. It covers the fundamental concepts of optical waveguides, including their classifications, characteristics, advantages, and disadvantages. We have also explored the behavior of propagating waves within waveguides to understand better how they behave when propagating through straight or bent waveguides. To this end, we have discussed the mathematical models for both straight and bent waveguides. Furthermore, we have provided an overview of the literature survey on optical waveguides.

- **Chapter 2:** This chapter establishes the understanding of the mathematical analysis of the bent waveguide model developed in the coming chapters. We describe the spectral theory of operators and the recent work done in the direction of the non-self-adjoint operators.

- **Chapter 3:** This chapter discusses the challenges encountered in analyzing the bent waveguide model, particularly about the non-self-adjointness of the governing problem. It is shown that the bent waveguide eigenvalue problem is non-self-adjoint. It has non-real eigenvalues. The imaginary part of the bent mode propagation constant indicates the amount of energy loss caused by the curvature of the waveguide. We decompose the operator involved in the eigenvalue problem into self-adjoint and non-self-adjoint parts. Furthermore, depending on the bent radius parameter, we demonstrate how the bent waveguide model transforms into a straight waveguide. This transformation turns the non-self-adjoint problem into a self-adjoint problem, resulting in real eigenvalues instead of non-real ones.

- **Chapter 4:** The analysis done in Chap. 3 poses questions on the behavior of the bent waveguide eigenvalue problem. Further mathematical analysis was done, which revealed that eigenfunctions corresponding to distinct eigenvalues are orthogonal. The relationship between the real and imaginary parts of the non-real eigenvalues is discussed. The asymptotic analysis of these modes proved that as $r \to \infty$ the behavior of the eigenfunctions is proportional to $1/\sqrt{r}$.
- **Chapter 5:** This is the last chapter of the thesis, which contains the conclusion of the thesis.

1.8 RESEARCH OUTCOMES

1.8.1 Publications

The following publications have accompanied the work presented in this thesis.

1. R. Kumar and K. R. Hiremath, *Non-self-adjointness of bent optical waveguide eigenvalue problem*, Journal of Mathematical Analysis and Applications, 512, (1), 126024, Elsevier, 2022.

2. R. Kumar, and K. R. Hiremath, *Mathematical analysis of bent optical waveguide eigenvalue problem*, Journal of Physics A: Mathematical and Theoretical - IOPscience, 57(34), 2024.

3. R. Kumar, K. R. Hiremath, and S. Manzetti, *A primer on eigenvalue problems of non-self-adjoint operators*, Analysis and Mathematical Physics, 14(21), Springer, 2024

1.8.2 Conferences and Workshops

- 34th International Workshop on Operator Theory and its Applications (IWOTA), University of Helsinki, Finland, July 31 - August 4, 2023.
  Talk Title: *Operator analysis of non-self-adjoint bent waveguide problem*

- Mini-courses in Mathematical Analysis, Edificio Paolotti, Via Paolotti 2/a, Padova, Italy, June 19 - 23, 2023.
  Talk Title: *Mathematical analysis of non-self-adjoint bent waveguide problem*

- National Symposium on Mathematics and Applications (NSMA), Department of Mathematics, Indian Institute of Technology Madras, India, 22nd December 2022.
  Talk Title: *Analysis of non-self-adjoint bent waveguide eigenvalue problem*

- International Conference on Mathematical Analysis and Applications (ICMAA), Department of Mathematics, National Institute of Technology, Tiruchirappalli, India, 15th-17th December 2022.
  Talk Title: *Analysis of non-self-adjoint bent waveguide eigenvalue problem*

- BIRS workshop on Mathematical aspects of the physics with non-self-adjoint operators Banff, Canada, 10th - 15th July, 2022.
  Talk Title: *Mathematical analysis of non-self-adjoint eigenvalue problem for the bent waveguides*

- Optics and Photonics Centre, Indian Institute of Technology Delhi, Summer School on Optics and Photonics (Jointly organized with National Academy of Sciences, India (NASI)-Delhi) 6th-24th June 2022.
  Talk Title: *Mathematical aspects of the bent waveguides*

- Workshop on Perturbation of Spectral Bands and Gaps, the Technical University of Dortmund (virtual), Germany, 26th -30th July 2021.

- International Workshop on Operator Theory and its Applications (IWOTA, virtual), Lancaster University, U.K, 16th - 20th August 2021.

- International Conference on Advanced Research in Applied Sciences (virtual), Kurukshetra University, Kurukshetra, India, 25th September 2021.
  Talk title: *Mathematical aspects of non-self-adjoint bent waveguide problem*

- International Conference on Advances in Differential Equations and Numerical Analysis (ADENA

virtual), Indian Institute of Technology Guwahati, India, 12th-15th October 2020.
Talk Title: *Mathematical aspects of non-self-adjoint bent waveguide problem*

1.9 CONCLUDING REMARKS

This chapter discussed waveguide models for straight and bent waveguides. The straight waveguide model has been studied extensively, but not the bent waveguide model. To get further insights into light propagation in bent waveguides, we have explored the bent waveguide model in this work. In this chapter, we have reviewed the basics of the waveguides and the literature survey on straight and bent waveguides. The straight and bent waveguide model and their mathematical construction are discussed, which is a foundation for the discussions in the coming chapters.