3
Extraction of Constellation Signature

3.1 INTRODUCTION

As explained in the previous chapter, the likelihood-based classifier requires a perfect channel model for optimal accuracy. If some channel parameters are unknown, then they are estimated by some means, increases the computational complexity. These classifiers depend on the accuracy of the estimated channel parameter and the matching model. Hence these methods are not useful in the blind environment and for real-time classification. Machine learning-based classifiers were developed for reliable performance with lower complexity. Supervised learning-based ML classifiers require a training phase to set the thresholds or weights for the final model to detect modulation type. Various strategies for classification in the thesis are unsupervised learning-based methods that work in the blind environment.

In the present approach, a blind signal is considered with all the parameters unknown. Carrier frequency, symbol rate, carrier frequency offset, phase offset, and symbol time offset are estimated, and offsets are corrected. The carrier frequency of the passband signal is estimated, and the RF signal is downconverted to zero center. The symbol rate of the signal is estimated, and the matched filtered signal is downsampled to one sample per symbol for extraction of constellation points.

3.2 CONSTELLATION EXTRACTION

The methods developed in this thesis use constellation points as an input for modulation classification. As constellation signature form clusters in the I/Q plane, different distance and density-based clustering algorithms are used to identify the modulation order and domain (ASK, PSK, and QAM). For use in practical purposes, root raised cosine pulse shaping has been assumed at the transmitting end. At the receiving end, the signal is sampled with a high sampling frequency to satisfy the Nyquist criterion. Figure 3.1 shows the block diagram of the system to extract constellation points. The model considered for complex waveform is given by

$$r(t) = x_p(t) + w(t) \qquad 0 \leq t \leq NT_s \tag{3.1}$$

Here symbol time is represented by T_s, and N is total number of symbols. $r(t)$ is the passband signal received after passing through AWGN environment represented by $w(t)$. The Probability distribution function of $w(t)$ is given by $G(z)$

$$G(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(z-\mu)^2}{2\sigma^2}} \tag{3.2}$$

Gaussian noise depends on two parameters that are mean, represented by μ, and standard deviation, represented by σ. The transmitted signal $x_p(t)$ with PSK and QAM can be denoted as:

$$PSK : x_p(t) = \sum_k Re[p(t - kT_s) e^{j\frac{2\pi}{M}(m-1)} e^{j2\pi f_c t}] \tag{3.3}$$

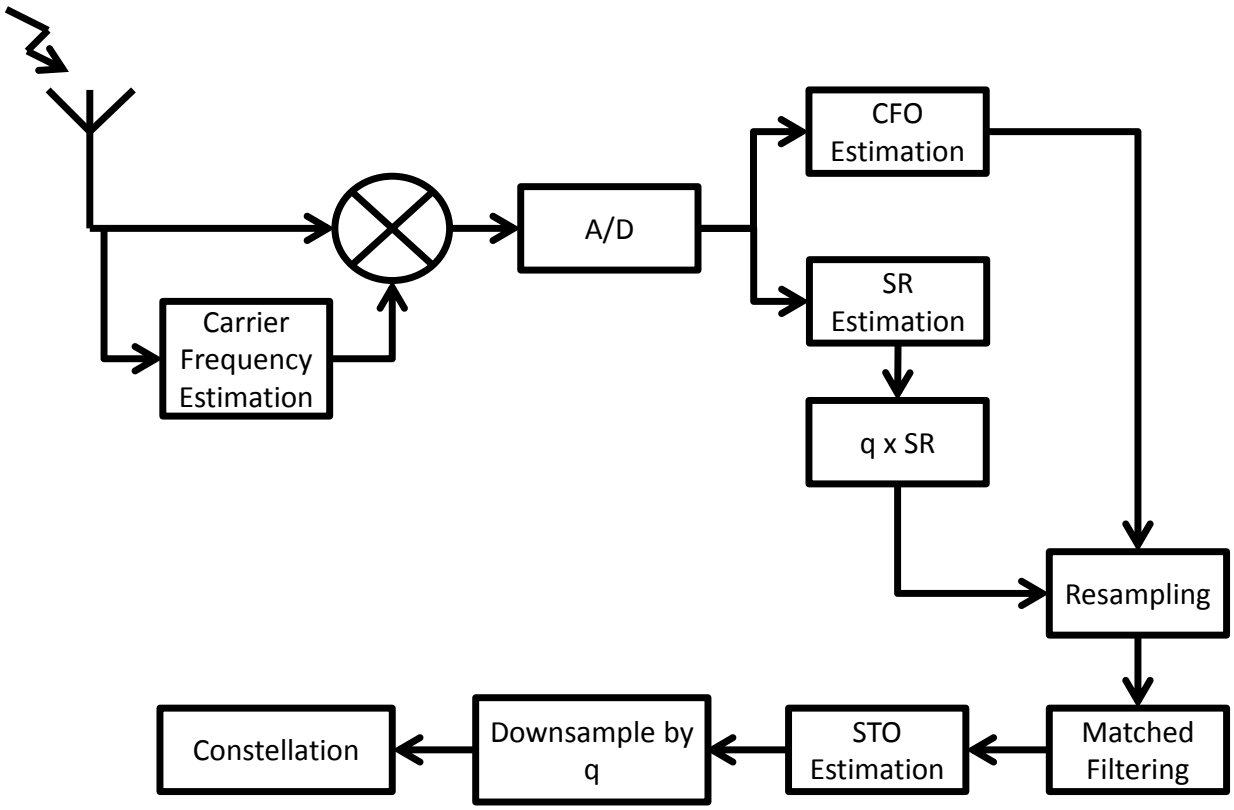

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Figure 3.1 : System model for extraction of constellation from received signal

$$QAM : x_p(t) = \sum_k Re[(a_k + jb_k)p(t - kT_s)e^{j2\pi f_c t}] \tag{3.4}$$

The received RF signal has a carrier frequency of f_c and the transmitted signal is pulse shaped with $p(t)$ of duration T_s. Modulation order is given by M and $m = 1, 2, ..., M$. Different orders of ASK, PSK, and QAM are considered as they are widely used in various modem and television applications. If the signal $r(t)$ is downconverted without frequency error, downconverted signal $y(t)$ can be expressed as:

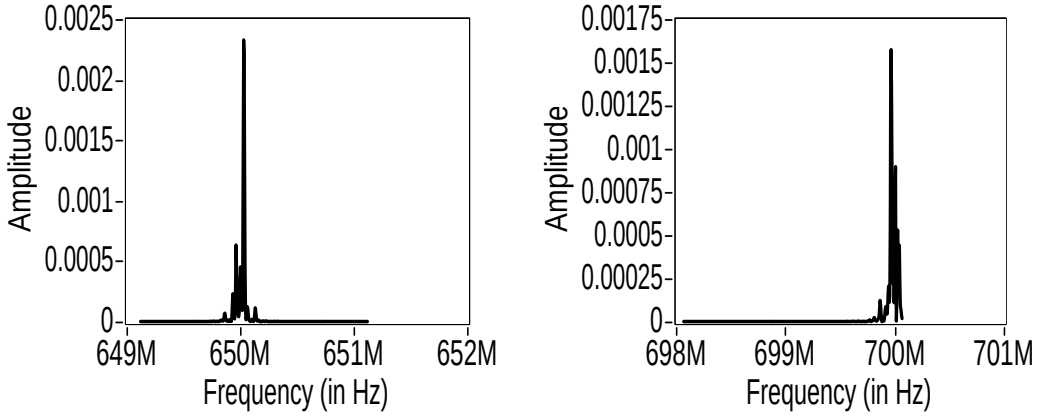
$$y(t) = r(t)e^{-j2\pi f_c t} \tag{3.5}$$

$y(t)$ is digitized with frequency f_s and expressed as $y[n]$.

$$PSK : y[n] = \sum_k Re[p(\frac{n}{f_s} - kT_s - \zeta T_s)e^{j\frac{2\pi}{M}(m-1)}] + w(\frac{n}{f_s} - \zeta T_s) \tag{3.6}$$

$$QAM : y[n] = \sum_k Re[(a_n + jb_n)p(\frac{n}{f_s} - kT_s - \zeta T_s)] + w(\frac{n}{f_s} - \zeta T_s) \tag{3.7}$$

To extract the constellation from the downconverted and digitized baseband signal, different parameters and offsets of the signal viz. carrier frequency offset (CFO), symbol rate, phase offset, and symbol timing offset (ζ) need to be estimated, and offsets need to be corrected.



(a) Spectrum of RF signal with carrier frequency 650 KHz.

(b) Spectrum of RF signal with carrier frequency 700 kHz.

Figure 3.2 : Spectrum of RF signal.

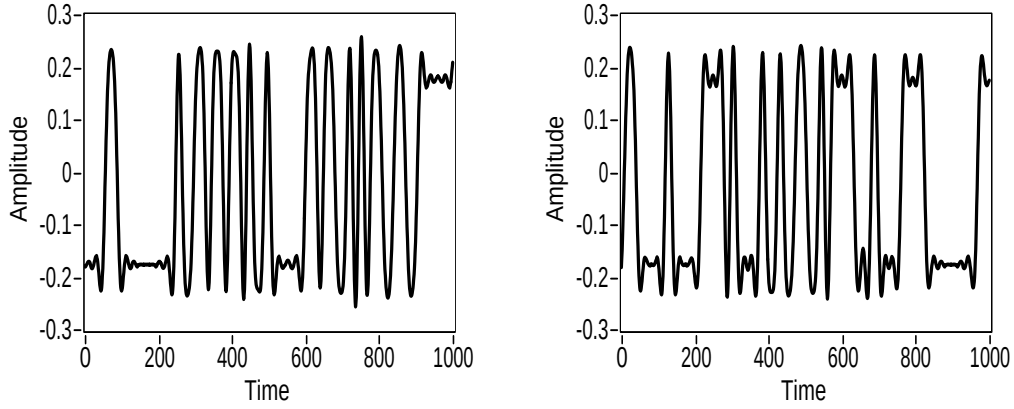


Figure 3.3 : Complex Baseband Pulse with Root Raised Cosine pulse shaping and roll off factor equals 0.5 for QPSK (a) In-Phase component (b) Quadrature phase component

3.2.1 Carrier Frequency Estimation

The intercepted passband signal is downconverted to a baseband signal using a carrier frequency of RF signal. Carrier frequency is estimated using the power spectrum of the signal. Linear modulations are considered whose spectrum is symmetrical. The bandwidth of the frequency band that contains 90% of the total power of the full span is found. The middle frequency corresponding to that band is the estimated carrier frequency with which the signal is downconverted. Figure 3.2 shows the spectrum of received RF signals with different carrier frequencies. Inphase and Quadrature phase waveforms of the complex baseband signal are given in Figure 3.3. If f_c is the actual carrier frequency and f_c' is the estimated carrier frequency with which RF signal is downconverted, the difference between actual and estimated carrier frequency is called CFO, which is discussed in the next section.

3.2.2 Carrier Frequency Offset Estimation and Correction

Carrier frequency offset in the signal is found using [Sethi and Ray, 2013]. If there is a mismatch between an actual carrier frequency (f_c) and estimated frequency of carrier (f_c'), the signal is downconverted to Δf_c, which is CFO and equal to $|f_c - f_c'|$. Due to CFO, constellation points rotates in I/Q plane. Figure 3.4(a) and 3.4(c) shows the rotated constellation of PSK and QAM when CFO is not corrected. Figure 3.4(b) and 3.4(d) shows the constellation structure when CFO is corrected. The transmitted signal $x_p(t)$ can be represented by

$$x_p(t) = x(t)e^{j2\pi f_c t} \tag{3.8}$$

After the signal is traveled through the AWGN channel, it is given by $r(t)$. The downconverted signal is denoted by $y(t)$, and sampled signal is denoted by $y(n)$.

$$r(t) = x(t)e^{j2\pi f_c t} + w(t) \tag{3.9}$$

$$y(t) = x(t)e^{j2\pi f_c t}e^{-j2\pi f_c' t} + w'(t) \tag{3.10}$$

$$y(t) = x(t)e^{j2\pi \Delta f_c t} + w'(t) \tag{3.11}$$

$$y(n) = x(n)e^{j2\pi \Delta f_c \frac{n}{f_s}} + w'(\frac{n}{f_s}) \tag{3.12}$$

$y[n]$ has the phase of $2\pi\Delta f_c \frac{n}{f_s}$ and $y[n+1]$ has the phase of $2\pi\Delta f_c \frac{n+1}{f_s}$. Due to this change in phase of points with each sample, extracted constellation rotates. To estimate Δf_c, $y(n)$ is passed through eighth order non-linearity, and the spectrum using FFT is calculated.

$$y(f) = |FFT(y^8(n))| \tag{3.13}$$

$y(f)$ is observed with the peak at a frequency of $8\Delta f_c$. This frequency is divided by 8 to get the actual CFO, and $y(n)$ is corrected. Figure 3.5 shows the graph of (3.13). Figure 3.5 is obtained by taking FFT of 8288 points of $y^8[n]$ with a sampling frequency of 1600 kHz and CFO of 10kHz. A peak at index 414 is observed, which corresponds to frequency 79922.77 Hz, i.e.,

$$8\Delta f_c = \frac{k f_s}{N} \tag{3.14}$$

$$8\Delta f_c = \frac{414 \times 1600k}{8288}$$

$$\Delta f_c = 9990.34Hz$$

$$y'(n) = y(n)e^{j2\pi(-\Delta f_c)\frac{n}{f_s}} \tag{3.15}$$

Here $y(n)$ can be represented in $in-phase$ and $quadrature-phase$ components for all linearly modulated signals. $2\pi\Delta f_c$ has been replaced with $\Delta\omega$. Signal after correction of CFO is given by (3.18).

$$y[n] = I[n] + jQ[n] \tag{3.16}$$

$$y'[n] = (I[n] + jQ[n])(\cos\frac{n\Delta\omega}{f_s} - j\sin\frac{n\Delta\omega}{f_s}) \tag{3.17}$$

$$= I[n]\cos n\frac{n\Delta\omega}{f_s} + jQ[n]\cos\frac{n\Delta\omega}{f_s} - jI[n]\sin\frac{n\Delta\omega}{f_s} + Q[n]\sin\frac{n\Delta\omega}{f_s}$$

$$y'[n] = I[n]\cos\frac{n\Delta\omega}{f_s} + Q[n]\sin\frac{n\Delta\omega}{f_s} + j(Q[n]\cos\frac{n\Delta\omega}{f_s} - I[n]\sin\frac{n\Delta\omega}{f_s}) \tag{3.18}$$

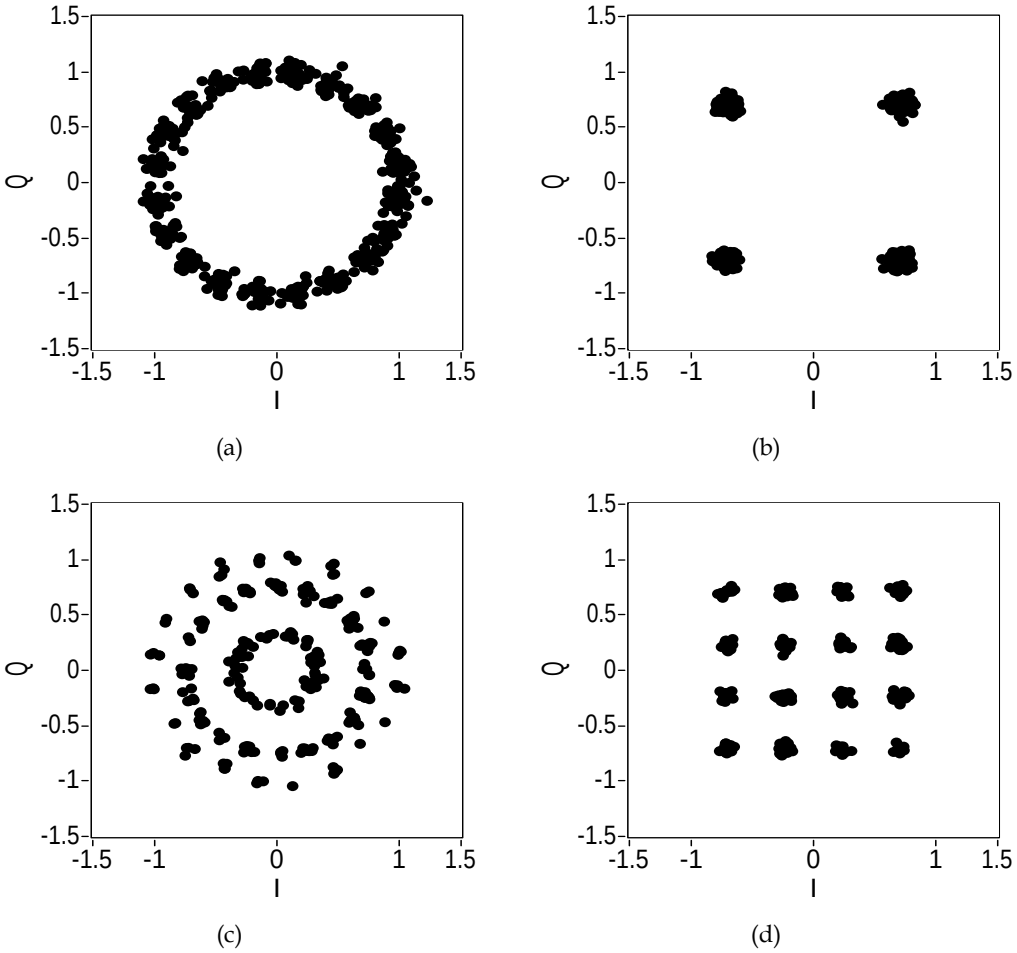


Figure 3.4 : Constellation of QPSK (a) with CFO (b) after CFO correction. Constellation of 16QAM (c) with CFO (d) after CFO correction.

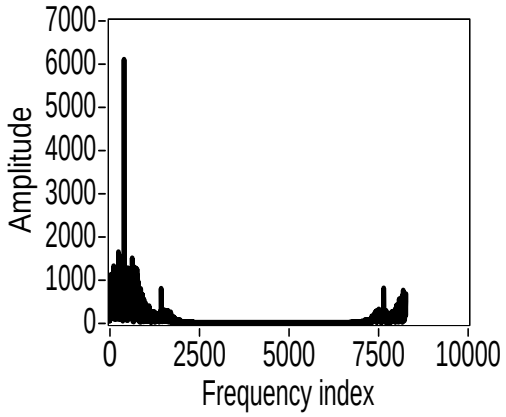


Figure 3.5 : Spectrum of eighth order non linearity

3.3 SYMBOL RATE ESTIMATION

After correcting the carrier frequency offset from the downconverted signal, the signal has to be downsampled to symbol rate (SR) or one sample per symbol for extracting constellation points. For that, the symbol rate has to be estimated. SR is estimated using two methods: one method is based on the complex envelope of the baseband signal. This method can estimate SR with CFO simultaneously [Sethi and Ray, 2013]. Another method is based on the instantaneous phase of baseband signal, which estimates SR after CFO correction [Jajoo *et al.*, 2019].

3.3.1 Symbol rate estimation based on complex envelope

Baseband signal $x(t)$ with CFO and phase offset is represented by $y(t)$

$$y(t) = x(t)e^{j(2\pi f_0 t + \theta_c)} \tag{3.19}$$

$x(t)$ has in-phase and quadrature phase component i.e.,

$$x(t) = x_I(t) + jx_Q(t) \tag{3.20}$$

$$x(t) = \sum_n a_n p(t - nT_s) + j\sum_n b_n p(t - nT_s) \tag{3.21}$$

$$c(t) = E(x_I^2(t) + x_Q^2(t)) = (a^2 + b^2)\sum_n p^2(t - nT_s) \tag{3.22}$$

Where a_k and b_k are such that $E[a_k a_j] = a^2\delta_{kj}$ and $E[a_k] = 0$ and $E[b_k b_j] = b^2\delta_{kj}$ and $E[b_k] = 0$. $\delta_{kj} = 0$ for $k \neq j$ and 1 if $k = j$.

$$C(\omega) = \mathscr{F}[(a^2 + b^2)\sum_n p^2(t - nT_s)] \tag{3.23}$$

$$C(\omega) = (a^2 + b^2)\sum_n \mathscr{F}[p^2(t) \circledast \delta(t - nT_s)] \tag{3.24}$$

$$= (a^2 + b^2)\mathscr{F}[p^2(t)]\sum_n \mathscr{F}[\delta(t - nT_s)]$$

$$= \frac{(a^2 + b^2)}{2\pi}[P(\omega) \circledast P(\omega)]\mathscr{F}[\sum_n \delta(t - nT_s)]$$

$$= \frac{(a^2 + b^2)}{2\pi}[P(\omega) \circledast P(\omega)]\sum_n \frac{2\pi}{T_s}\delta(\omega - n\frac{2\pi}{T_s})$$

$$= \frac{(a^2 + b^2)}{T_s}[P(\omega) \circledast P(\omega)]\sum_n \delta(\omega - n\frac{2\pi}{T_s}) \tag{3.25}$$

$\mathscr{F} < . >$ represents Fourier transform and $(\alpha \circledast \beta)$ represents convolution between α and β. $C(\omega)$ has peaks at multiples of the symbol rate. As $P(\omega)$ is bandlimited, the amplitude of peaks for $n \geq 2$ has very small values. The above expression of $C(\omega)$ is followed by ASK, PSK, and QAM while in FSK, no such peak is observed. Figure 3.6 is the spectrum of the complex envelope for the signal with the same parameters used in CFO estimation. The peak is observed at frequency index 507, which corresponds to the symbol rate of 100 kHz.

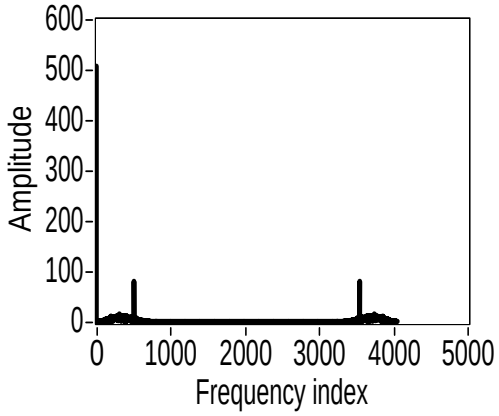


Figure 3.6 : Spectrum of squared complex envelope

3.3.2 Symbol rate estimation based on instantaneous phase

A new method for symbol rate (SR) estimation has been proposed, which is independent of phase offset. As PSK and QAM signals have a phase change at every symbol duration, the spectrum of the instantaneous phase after passing through the average filter shows a peak at the symbol rate. The phase of a complex baseband signal with rectangular pulse shaping has discrete phase levels. Let $\phi[k]$ be the phase represented by:

$$\phi[k] = \sum_{n=1}^{N} A_n p(k - nT_s) \tag{3.26}$$

In Equation (3.26), A_n is discrete phase levels that change at every symbol duration T_s. For a pulse shaping filter other than rectangular such as raised cosine or RRC, $p(.)$ take the values accordingly. Figure 3.7 is obtained for QPSK with a sampling frequency of 160 kHz, symbol rate of 10 kHz, and RRC pulse shaping. In Figure 3.7(a), a, b, d, e are constellation points, and c, c', f are different instances of transition between symbol points. Fig. 3.7(b) shows the instantaneous phase ranging from $[-180^0 180^0)$, which has abrupt changes when the transition occurs from $c \leftrightarrow c'$. Other abrupt changes come, when transition follows a path $d \leftrightarrow f \leftrightarrow a$ where angle changes from $(-135^0 \leftrightarrow 45^0)$. A moving average filter is used to smoothen these abrupt changes, and $\phi'[k]$ is obtained as shown in Figure 3.7(c) and represented by

$$\phi'[k] = \sum_{n=1}^{N} A_n p'(k - nT_s) \tag{3.27}$$

Squaring 3.27 and suppressing the DC term, $q[k]$ is obtained shown in Figure 3.7(d).

$$q[k] = \phi'^2[k] - E(\phi'^2[k]) \tag{3.28}$$

$Q(\omega)$ is calculated by taking the Fourier transform (represented by $\mathscr{F}$) of $q[k]$.

$$Q(\omega) = \mathscr{F} \sum_{n} [(A_n^2) p'^2(k - nT_s)] - \mathscr{F}[E(\phi'^2[k])] \tag{3.29}$$

Here k represents $E(\phi'^2[k])$.

$$Q(\omega) = \mathscr{F} \sum_{n} [(A_n^2) p'^2(k - nT_s)] - K \tag{3.30}$$

$$Q(\omega) \overset{(a)}{=} \sum_n A_n^2 \mathscr{F}[p'^2(k) \circledast \delta(k - nT_s)] - K] \tag{3.31}$$

$$\overset{(b)}{=} \sum_n A_n^2 \mathscr{F}[p'^2(k)] \mathscr{F}[\delta(k - nT_s)] - K \tag{3.32}$$

$$\overset{(c)}{=} \sum_n A_n^2 [P'(\omega) \circledast P'(\omega)] \frac{2\pi}{T_s} \delta(\omega - n\frac{2\pi}{T_s}) - K \tag{3.33}$$

$$Q(\omega) = \frac{2\pi}{T_s} [P'(\omega) \circledast P'(\omega)] \sum_{\forall n, n \neq 0} A_n^2 \delta(\omega - n\frac{2\pi}{T_s}) \tag{3.34}$$

Here $\circledast$ represents convolution. (a) follows from the fact that $h[n] \circledast \delta[n-k] = h[n-k]$. (b) and (c) follows from convolution and multiplication property of Fourier transforms. Here K is equivalent to $Q(\omega)$ for $n = 0$. $Q(\omega)$ is shown in Figure 3.7(e).

For validation of peak in $Q(\omega)$, the length of the moving average filter is varied, and corresponding to each length, the sharpness of peak (G_l) is calculated using (3.35).

$$G_l = \frac{\sum_{k=i-100}^{i+100} (max(Q_l[f]) - Q_l(k))}{200} \tag{3.35}$$

Here i is the index value, corresponding to the maximum of $Q_l(f)$ for a filter length l. The frequency corresponding to a maximum value of G_l for any l is estimated as SR. Signal $y[n]$ given in (3.18) is used for constellation extraction using estimated SR. Extracted constellation is normalized to zero mean and unit power.

After symbol rate estimation, the signal is resampled to integer (let it be q) times symbol rate for matched filtering. As root raised cosine pulse shaping is considered, at the receiver side, FIR filter with coefficients corresponding to root raised cosine pulse shaping and q sample per symbol is set. After matched filtering, the symbol timing offset is estimated and corrected.

3.3.3 Symbol time offset estimation

Matched filtered signal has a timing offset i.e., from those q samples/symbol, it is unknown that which samples correspond to constellation points. To estimate timing offset from total N samples, the signal is bunched into N/m number of sets where each bunch has q values. i_{th} value corresponding to each bunch is added (where i ranges from 1 to q), and the outcome will be an array of q samples from which the index corresponding to the maximum value is an estimate of symbol timing offset. If l is desired index, every $(l + rq)^{th}$ sample is taken, where $r = 0, 1, 2, q$ such that $(l + kq)^{th}$ data point corresponds to the last acquired constellation point. Figure 3.8 shows the bunches B_1, B_2,...$B_{\frac{N}{q}}$ by dividing data $y[i]$ into groups of q. After downsampling the matched filtered waveform, constellations obtained for different modulations are shown in Figure 3.9.

3.3.4 Phase offset estimation

In the case of slow and flat fading, the constellation rotates with an angle θ_0 for single signal realization. Each CFO corrected constellation point $d[n]$, has its own phase with an extra phase of θ_0 ($\angle\alpha = \theta_0$). The phase offset of PSK and QAM are estimated using different methodology as explained.

For QAM modulation

If the modulation domain is identified as QAM in the algorithm hierarchy, the phase offset is identified using [Cartwright, 1999]. Let the received complex baseband symbol is represented by d. Then with phase offset and AWGN noise, it is represented by

$$d = e^{j\theta_0} s + w \tag{3.36}$$

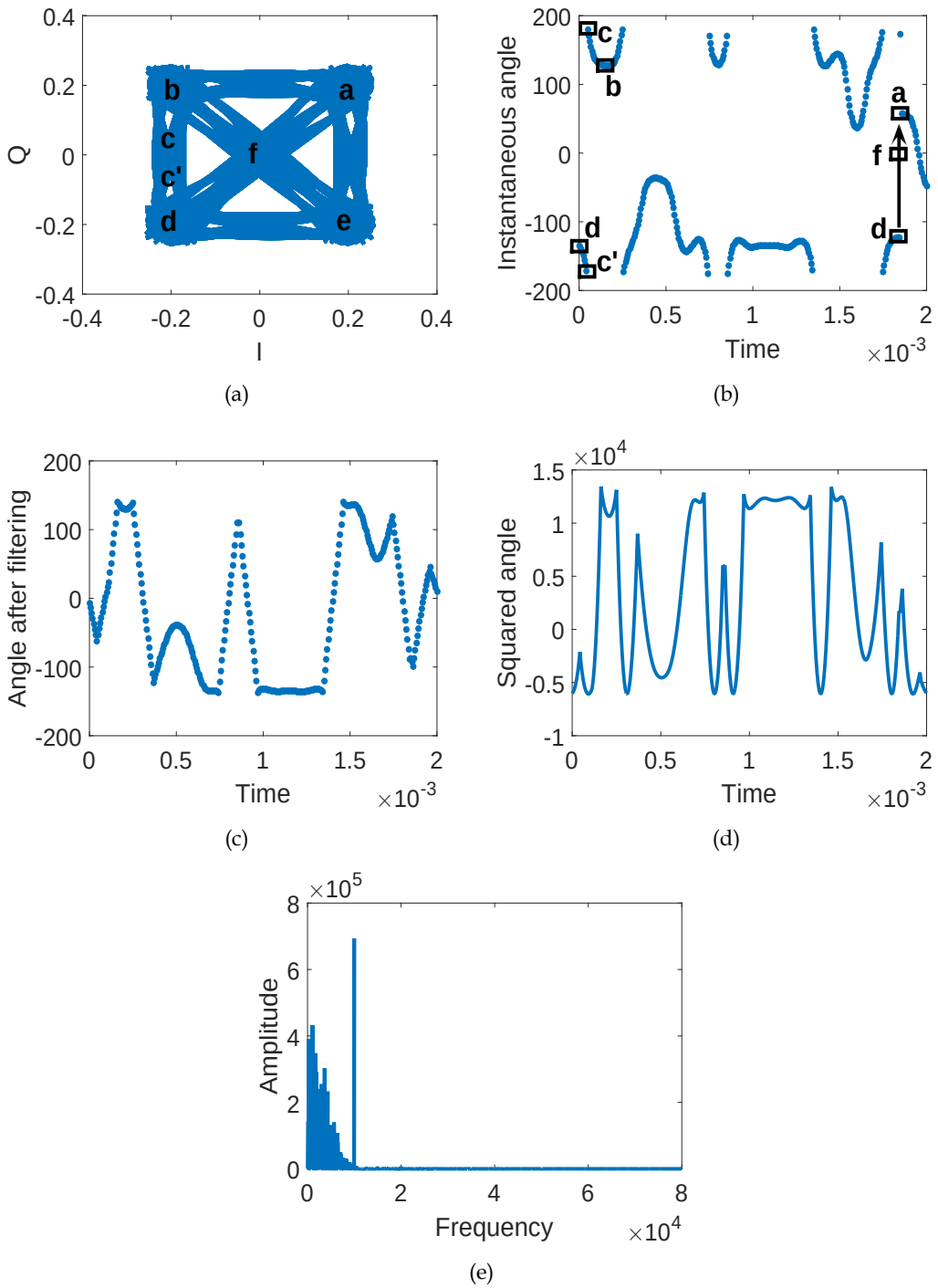


Figure 3.7 : (a) Root raised cosine pulse-shaped QPSK constellation with transitions. (b) Instantaneous angle of 3.7(a). (c) Phase after passing 3.7(b) through moving average filter of length 9. (d) Graph obtained after squaring and shifting 3.7(c) to 0 mean. (e) Spectrum of 3.7(d) with a peak at a symbol rate of 10 kHz.

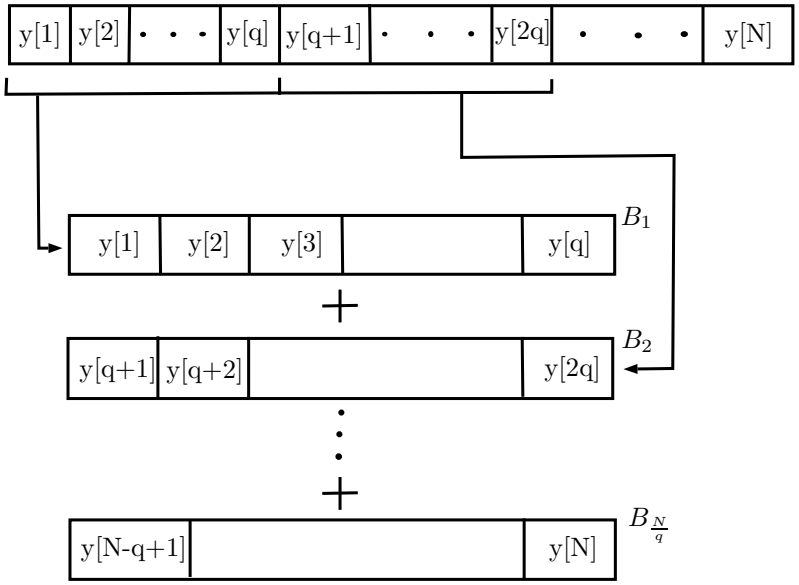


Figure 3.8 : Symbol timing offset estimation visualization

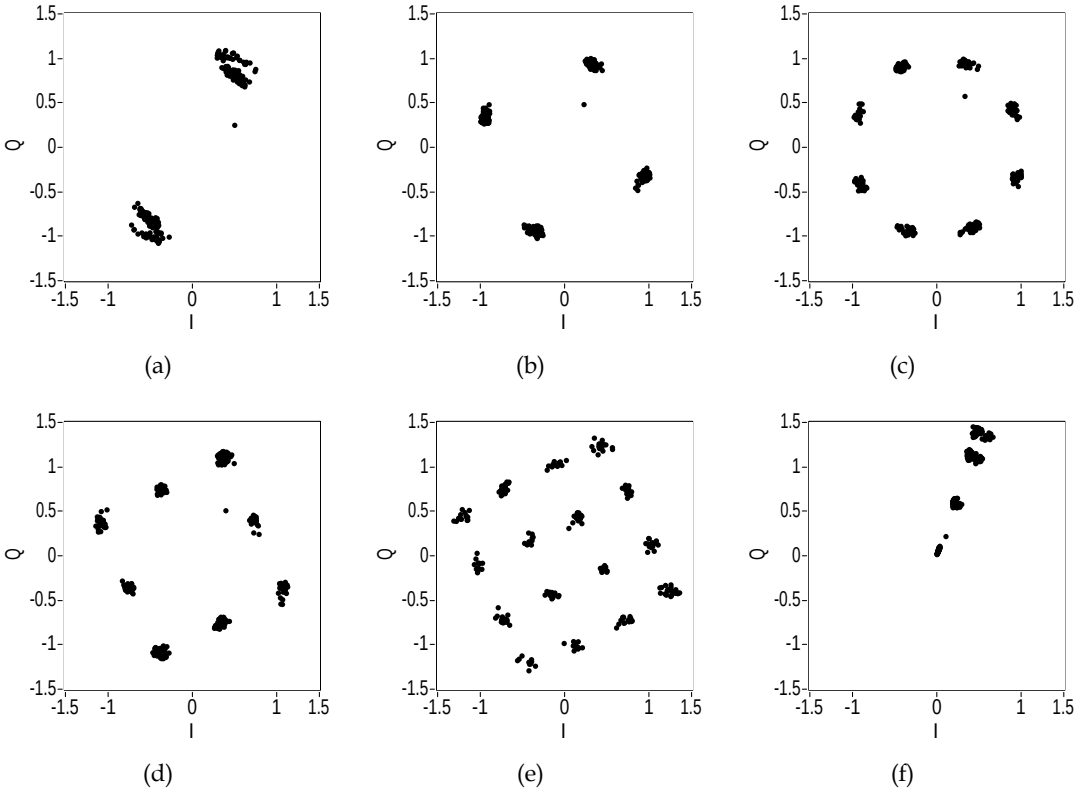


Figure 3.9 : Constellation points obtained for (a) BPSK (b) QPSK. (c) 8PSK (d) 8QAM (e) 16QAM (f) 4ASK.

Here the real part and the imaginary part of d are represented by d_r and d_i, respectively. Similarly, the real and imaginary part of transmitted complex baseband symbol s are represented by s_r and s_i, respectively. s_r and s_i are considered independent.

$$\gamma = E[d_r^4] + E[d_i^4] - 6E[d_r^2 d_i^2] \tag{3.37}$$

$$= \gamma_s cos 4\theta_0$$

$$\gamma_s = E[s_r^4] + E[s_i^4] - 6E[s_r^2 s_i^2] \tag{3.38}$$

$$\gamma_a = E[d_r^3 d_i] - 3E[d_r^2]E[d_r d_i] \tag{3.39}$$

$$\gamma_b = E[d_i^3 d_r] - 3E[d_i^2]E[d_i d_r] \tag{3.40}$$

$$\gamma_a - \gamma_b = .25\gamma_s sin 4\theta_0 \tag{3.41}$$

$$tan 4\theta_0 = 4\frac{\gamma_a - \gamma_b}{\gamma} \tag{3.42}$$

This method does not work for PSK modulations, hence a new method is proposed for phase offset estimation.

For PSK modulation

$$d[n] = |\alpha| e^{j\theta_0} s[n] \tag{3.43}$$

Sum of all arguments of $d[n]$ is defined by ψ.

$$\psi = \sum_{n=1}^{N} arg(d[n]) \tag{3.44}$$

$$\psi = (\theta_1 + \theta_0) + (\theta_2 + \theta_0) ... + (\theta_N + \theta_0) \tag{3.45}$$

$$\psi = \sum_{i=1}^{N} \theta_i + N\theta_0 \tag{3.46}$$

As constellation has symmetric structure, $\frac{1}{N}\sum \theta_i \approx 0$ for large number of equiprobable symbols. Hence, approximate value of θ_0 is given by

$$\theta_0 \approx \frac{\psi}{N} \tag{3.47}$$

The phase offset from the extracted constellation signature is corrected, and different clustering and regression approaches are used for modulation classification explained in further chapters.

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