

Investigating the effect of deliberate refractive index disorder and presence of complex-coupling in specialty topological lattice

In this chapter we investigate and establish the fact that, a topological interface-state and transverse localized state can coexist due to the interplay of band topology and deliberate refractive index disorder in one-dimensional photonic lattice. The specialty topological lattice consists of two periodic lattices of dissimilar topology as described in the previous chapter. The topological nature of the lattices are identified by the associated *Zak* phases of the corresponding bands. We found that two exclusive states of light are supported by a 10 mm long specialty topological lattice in presence of optimum disorder. The signature of the topological interface-state is identified by its robust loss-less nature whereas transverse localized state is identified by the exponentially decaying tail of the transverse intensity distributions. The studies in this direction are necessary to explore novel light guiding schemes which can enhance the light guiding capacity in one-dimensional photonic lattices as well as are robust to the fabrication imperfections. Further, the effect of complex-coupling is also investigated. It is found that robust amplification of the topological interface state is possible by introducing optimum gain to the substrate of the specialty topological lattice. By numerical simulation, it is illustrated that these states can be amplified without any distortion by introducing homogeneous gain to the lattice substrate. This is a strikingly different phenomenon from the conventionally known fact that with the introduction of gain to periodic lattices, the propagating state of light spreads in the transverse direction due to enhanced complex coupling. The findings suffice to propose the utility of such systems as on-chip optical interconnects and have the potential for topological amplifiers within the regime of optical communication.

5.1 Introduction

The discovery of integer quantum hall effect in a two dimensional electron system gave birth to the domain of topological physics in the year 1980 [Klitzing et al., 1980]. Later in 1988, the theoretical idea of achieving topological phenomenon in periodic lattice was presented [Haldane, 1988]. Finally, the idea was conceived in the photonic regime by Raghu and Haldane [Raghu and Haldane, 2008] and was experimentally verified by Wang using electro-magnetic waves [Wang et al., 2009]. Topological photonics ensure various exotic design mechanism and

functionalities that are robust against disorder and fabrication imperfections [Segev and Bandres, 2021; Yang et al., 2020a; Bandres et al., 2018]. Over the years, various experimental verification of topological phenomenon in two-dimension (2-D) and three-dimension (3-D) systems are confirmed by the existence of interface states [Khanikaev et al., 2013; Yang et al., 2019; Dutt et al., 2020]. Apart from these, special models were proposed to observe topological nature in one-dimension (1-D) lattice. It was obtained by customizing the band-topology via transition from trivial to non-trivial or vice-versa. During this transition, loss-less robust interface-state is formed at the vicinity of phase transition. This transition is reportedly achieved by Bose-Hubbard, Abry-André-Harper (AAH) and Su-Schrieffer-Heeger (SSH) models [Pinto et al., 2009; Liu et al., 2015; Zeng et al., 2020; Yahyavi et al., 2019]. Moreover, as a result of extensive research in this domain, the phenomenon of phase transition have also been observed in one-dimensional periodic lattices [Xiao et al., 2014; Zhu et al., 2018]. The topological nature is identified by *Zak* phase analysis. The value of *Zak* phase is crucial to determine the topological nature of the system. The value for the *Zak* phase for the bands can be either 0 or π . The conditions to determine the topological nature of a lattice from its band-structure is that, if the summation of all the *Zak* phases of the bands lying under the bands corresponding to the operational wavelength band is even multiple of π , then the lattice is topologically trivial but if the summation is odd multiple of π then the lattice is topologically non-trivial [Zhu et al., 2018].

On the other hand, the light guiding behavior of one-dimensional periodic photonic lattices in the presence of disorder is fascinating. The applied disorder traps the diffusive nature of the wave, resulting in transverse localization of light. P. W. Anderson proposed this theory stating that, the wave-function always gets confined to only a few sites under appropriate disorder [Anderson, 1958]. However, the basic assumption of the absence of interaction between the particle as well as the time invariance of the potential in the model were causing hindrance for its experimental observation. Later, scientists have shown that localization is fundamentally a wave based phenomenon and can be observed even in the optical regime [John, 1984]. Moreover, in optical regime, the photons does not interact with each other, as a result the study of localization in photonic platform is of major interest. Using this idea, experimental observations were finally carried out by D. S. Weirsmas in the photonic domain [Wiersma et al., 1997]. Gradually, the phenomenon of localization of light is experimentally observed in platforms like photorefractive crystals, polymeric binary fibers, disordered glassy fibers and laser written disordered glass wave-guides. The onset of localization in photorefractive crystal was studied and reported by Schwartz et. al. in the year 2007 [Schwartz et al., 2007]. Through this experiment, they proposed that in such platforms reshaping of disorder would be easier which can lead to the ideation for new class of switches. However, the method faced hindrance due to large disorder in grain size as well as refractive index mismatch. This fabrication bottleneck was overcome in the year 2012 by Karbasi et. al. [Karbasi et al., 2012]. They developed a binary fiber where micron scale disorder can be obtained easily. The fabrication technique of the fibers are very straightforward which enables fiber drawing technology starting from a preform. It is demonstrated that such fibers can be utilized for wave front shaping, image transport, quantum communication and key distribution and controlled focusing. However, plastic components like polymethyl methacrylate (*PMMA*) and polystyrene were used as preform. As a result, these fibers were showing very high propagation loss at optical frequency. To overcome the high losses incurred by plastic fibers, Karbasi et. al. proposed glass-based binary fibers in the 2014 which can be fabricated from porous glass [Karbasi et al., 2014]. This glass-based binary fibers promised to overcome all the bottlenecks faced by plastic fibers but suffered from the effect of non-homogeneity in the introduced disorder. Finally in the year 2020, Gianfrate

et. al. proposed a technique of employing a direct laser writing on glass based wave-guides [Gianfrate et al., 2020]. Till now the method suffers from the effect of inefficient coupling which leads to small refractive index contrast during fabrication. Further investigations are required in this domain as light dynamics in the disordered system can lead to the discovery of unknown phenomena occurring in mesoscopic regime. Additionally, light dynamics in the disordered system is a subject of immense interest and technological importance with a variety of state-of-the-art applications like image transport, random lasing, spectroscopy as well as exhibition of topological phenomenon in 2-D lattices [Karbasi et al., 2014; Redding et al., 2012, 2013; Liu et al., 2020].

In this chapter we study the effect of deliberate refractive index disorder on the one-dimensional specialty topological lattice. As described in the previous chapter, the specialty lattice consists of two periodic sub-lattices having different topologies. The idea is derived from the fact that a periodic trivial lattice can be transformed to a topologically non-trivial lattice by appropriately choosing the width of high index layers [Bhattacharjee et al., 2021]. The topological nature of the lattices are identified by analyzing the *Zak* phase. On getting confirmation about the topological nature of both the lattices, they are attached side-by-side to design the specialty topological lattice. A Gaussian beam is launched at the junction of the two sub-lattices and it is found that the beam propagates with constant amplitude. The loss-less transmission establishes the fact that the state of light supported at the junction is an topological interface-state which is topological in nature. On the other hand, the role of deliberate disorder on the propagation dynamics of the interface-state is of great importance. It has been already demonstrated in 2-D topological lattice that the topological interface-state remains robust to the introduced defects and deliberate disorder [Khanikaev and Shvets, 2017; Yu et al., 2021]. However, the role of disorder in one-dimensional topological lattice has never been exploited. In this chapter we report a systematic study of the effect of deliberate disorder on the propagation dynamics of the topological interface-state. It is observed that the topological interface-state remain robust to the introduced disorder over a range of wavelength dictated by the topological nature of the lattices. In contrast, it is widely known that deliberate disorder above the threshold level induces transverse localization. Since, our study deals with the topological lattices where there is no variation of the refractive index along the direction of the propagation of the light beam as well as the topological interface state is hosted at the junction of the two topologically opposite sub-lattices, it is fair to expect that transverse localized state would be hosted in them due to the application of deliberate refractive index disorder of right amount. However, it is found that only the trivial sub-lattice exhibits the phenomenon of transverse localization whereas its non-trivial counterpart remain robust to the applied disorder when the operational wavelength is not at the band-center. Therefore, by the introduction of optimum correlated disorder, it is observed that the topological interface-state remains unaffected but interestingly the trivial sub-section of the lattice started exhibiting the phenomenon of Transverse Anderson Localization (*TAL*). Thus, we experience two exotic light behaviors simultaneously. In this chapter this unique scheme of hosting transverse localized light state and topologically protected light state simultaneously in a single wave-guide lattice in descried. One of the light beams propagate through the topologically protected state occurring at the interface of the two topologically different lattices and the other beam propagates through the transverse localized state appearing in the trivial sub-lattice due to the presence of long-range correlated disorder. We believe that, further developments in this direction will pave the path for new light guidance schemes to facilitate inter-chip as well as intra-chip communication in all photonic systems.

Moreover, the effect of distributed as well as selective gain and loss judiciously integrated

to the designed specialty topological lattice is investigated to explore new phenomena as an interplay between topology and non-Hermiticity [Ota et al., 2020; Song et al., 2020; Silveirinha, 2019; Ao et al., 2020; Longhi, 2019; Rui et al., 2019; Song et al., 2019; Zhu et al., 2020; Wang et al., 2019b; Höckendorf et al., 2019]. Conventionally, the introduction of uniform gain to bulk results in transverse spreading of light owing to enhancement in complex coupling [Bhattacharjee et al., 2020]. However, the scenario is strikingly different for the topologically guided state of light. Instead of spreading, a robust amplification of the protected state of light at the end of a 7mm long specialty lattice is observed where a uniform gain is introduced only to the low-index bulk. These findings would soon lead to the design and realization of a specialty topological amplifier in an entirely periodic platform avoiding any complex spatial modulation of the lattice.

In this chapter we report the simultaneous hosting of *TAL* state and topological interface state due to the interplay of transverse refractive index disorder along with band-topology in one-dimensional specialty topological lattice. Additionally, robust amplification of the topological interface state due to the application of gain to the bulk of the lattice is also illustrated. In the first section of this chapter we present a brief literature review covering the domain of topological photonics, transverse localization of light and the effect of gain and loss in optical lattices. Though there are many reports addressing these domains of photonics individually, but their interplay have never been studied. In section II, the design of the specialty topological lattice and identification of loss-less interface state is reported. In section III, we investigate the effect of transverse refractive index disorder in topologically trivial lattice and topologically non-trivial lattice individually as well as on the specialty topological lattice designed by joining these trivial and non-trivial lattice side-by-side. It is observed that the topologically trivial lattice hosts transverse localized state of light due to the application of transverse refractive index disorder above a threshold value however, there is negligible effect of transverse refractive index disorder on the beam of light propagating through the topologically non-trivial lattice. By exploiting this concept two non-overlapping state of light corresponding to *TAL* state and topological interface state is hosted in the specialty topological lattice simultaneously. Further, simultaneous propagation of these two states through the specialty topological lattice is also reported. It is found that these two non-overlapping state of light propagate through the lattice without interacting with each other. In the next section, we study the light dynamics through gain and loss assisted specialty topological lattice. Effect of homogeneous as well as selective application of gain and/or loss to the specialty topological lattice is investigated. It is found that by judicious application of gain, the topological interface state can be amplified. Finally, in the last section we summarize by mentioning the major findings reported in this chapter and their probable device level application.

5.2 Topological lattice design and identification of loss-less interface state

One-dimensional coupled wave-guide lattice is a versatile test-bed to explore various exotic phenomenon involving light matter interaction. In this platform, band-topology of the lattice is tuned by optimizing the coupling strengths between the wave-guides. To optimize the coupling strength, refractive index contrast between the wave-guides may be varied or width of the wave-guides may be tuned. Here, we have varied the width of the wave-guides to realize two lattices having different band-topologies. The topological property is determined by analyzing

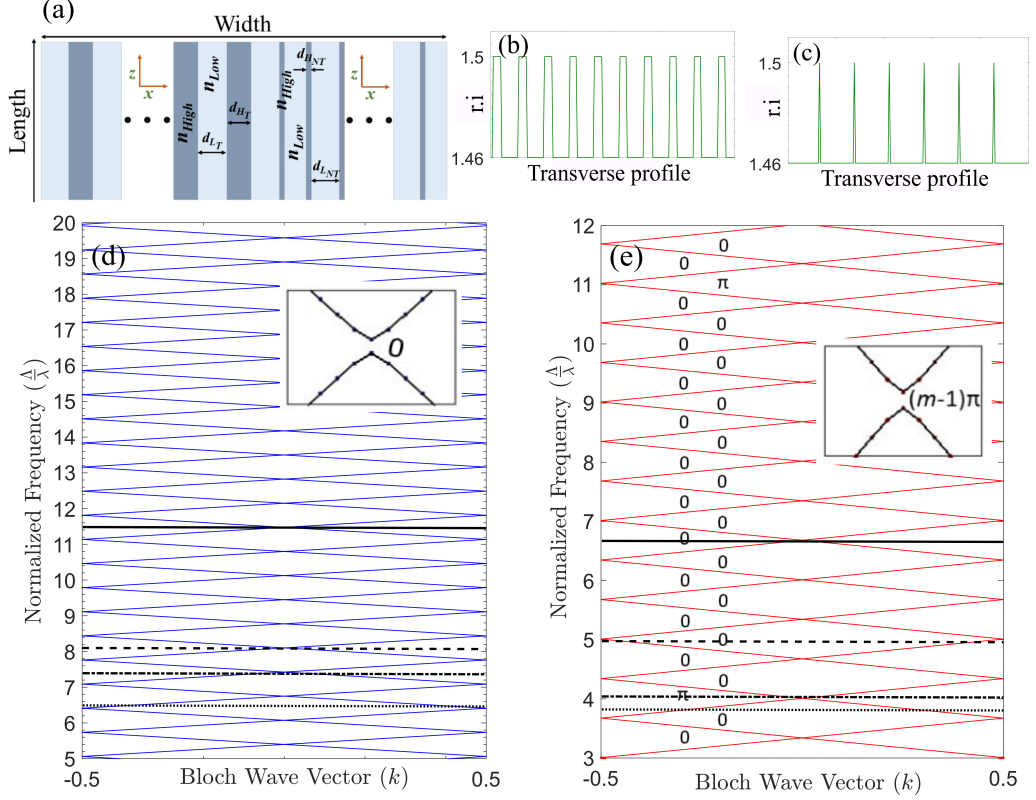


Figure 5.1: (a) Schematic of a periodic photonic lattice having refractive indices n_{High} and n_{Low} of width d_H and d_L respectively is presented. Variation of refractive index across the transverse direction of the trivial and non-trivial lattices are depicted by (b) and (c) respectively. The band structure over the approximate wavelengths ranging from $0.6 \mu\text{m}$ to $2.5 \mu\text{m}$ for the trivial and the non-trivial lattices are shown by plots (d) and (e) respectively where Λ depicts the lattice constant and λ denote the operating wavelength. The *Zak* phases of the respective bands of the non-trivial lattice changes between 0 and π as shown in (e) but the *Zak* phases of the respective bands of the trivial lattice does not changes between 0 and π for this lattice. The summation of the *Zak* phases of the bands lying under $1.8 \mu\text{m}$ and above $0.7 \mu\text{m}$ for the trivial and the non-trivial lattices are shown in the inset where m represents an even number.

the *Zak* phase. For one-dimensional periodic system *Zak* phase is expressed as [Xiao et al., 2014] :

$$\theta_n = \int_{-\pi/a}^{\pi/a} \left[i \int_0^a dx \epsilon(x) u_{n,k}^*(x) \partial_k u_{n,k}(x) \right] dk \quad (5.59)$$

where $\epsilon(x)$ is the transverse dielectric function, n denotes the number of energy band and $u_{n,k}(x)$ denotes the Bloch eigen-function of the state on the n^{th} band with wave vector k .

Schematic of the one-dimensional specialty topological lattice is depicted in Fig. 5.1 (a) where n_{High} and n_{Low} represent refractive indices of the materials of width d_H and d_L respectively. For fabrication feasibility, the lattices are assumed to be made from two materials having refractive index contrast of 0.04. For one of the lattices, the width of the periodic layer composed of lower refractive index material is $7\mu\text{m}$ and of higher refractive index material is $6\mu\text{m}$ whose transverse refractive index profile is shown in Fig. 5.1 (b). Similarly another

lattice is designed where the width of the layers composed of lower refractive index material is kept constant at $7\mu\text{m}$ but the width of the layers made of higher refractive index material is optimized to be $0.6\mu\text{m}$. The transverse refractive index profile of this lattice is shown in Fig. 5.1 (c). The detailed mechanism of optimizing the bi-layer dimension is described in the previous chapter. The sub-lattices are composed of 75 bi-layers having only the nearest neighbor coupling. The band-structure of the lattices are shown in Figs. 5.1 (d) and (e) respectively where Λ denote the lattice constants and λ denote the operating wavelength. The solid, dashed, dash-dotted and the dotted lines denote normalized frequency corresponding to four different wavelengths $1.125\mu\text{m}$, $1.55\mu\text{m}$, $1.8\mu\text{m}$ and $2\mu\text{m}$ in the Figs. 5.1(d) and (e) respectively. The propagation dynamics of a Gaussian beam will be studied in the next section at these wavelengths in presence and absence of refractive index disorder. This will demonstrate the approximate propagation characteristics through the specialty topological lattice at various operation regimes of the band-structure. To analyze the topological nature of these lattices, *Zak* phases of each of the bands are calculated by using Eq. 5.59. The *Zak* phases of all the bands for the lattice whose profile is shown in Fig. 5.1 (b) are 0. This result signifies that the lattice is topologically trivial. On the other hand for the lattice whose profile is shown in Fig. 5.1(c), there are changes of *Zak* phases from 0 to π as indicated in the 5.1(e). From this result we can infer that the lattice is topologically non-trivial in nature.

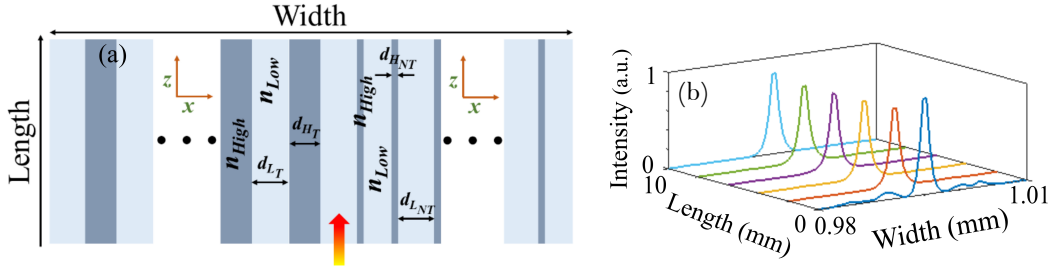


Figure 5.2: (a) Schematic of the specialty topological lattice is shown where the red arrow indicates the approximate location for launching of light beam. The refractive indices of the materials are denoted by n_{High} and n_{Low} where d_{HT} and d_{LT} represents the width of the layers corresponding to trivial sub-lattice and d_{HNT} and d_{LNT} represents the width of the layers corresponding to non-trivial sub-lattice. (b) The propagation dynamics of the input Gaussian beam has been shown, indicating loss-less nature of topologically protected state.

Eventually, after identifying the topological nature of the lattices, they are joined side-by-side to design the specialty lattice and the junction is excited with a Gaussian beam to establish the existence of the loss-less interface-state as described in the previous chapter. The combined specialty topological lattice consists of 150 bi-layers out of which 75 bi-layers have the dimension of trivial lattice and the other 75 bi-layers have the dimension of non-trivial lattice. The total lattice is 10mm long having a thickness on the order of the wavelength of the input light beam and have transverse width of 1.545 mm. The pictorial schematic of the specialty topological lattice is represented in Fig. 5.2(a) where the red arrow depicts the junction indicating the approximate location where the beam is to be launched. From Fig. 5.2(a) it can be understood that the trivial sub-section of the lattice is wider than the non-trivial sub-section as the width of each of the bi-layers of the trivial sub-section is $5.4\mu\text{m}$ more compared to the non-trivial section and both the sub-sections have equal number of bi-layers. Thus the trivial sub-lattice is 0.975mm wide and the non-trivial sub-lattice is 0.57mm wide. In the next section we would investigate the modification in the beam propagation dynamics through these trivial and non-trivial lattice in presence of deliberate refractive index disorder.

5.3 Investigating the effect of transverse refractive-index disorder

Transverse Anderson Localization (*TAL*) is a unique phenomenon occurring in periodic lattices due to the presence of right amount of disorder [Schwartz et al., 2007]. It is reported that for one-dimensional and two-dimensional periodic photonic lattices of infinite length, any small amount of disorder would lead to localization of light, however for three-dimensional systems of infinite length the disorder introduced should be above a certain threshold level to host the phenomenon of localization. Since we have considered a one-dimensional lattice of finite length, the disorder introduced should be above a threshold value for onset of *TAL*. Our aim was to study the effect of disorder on the propagation dynamics of the topological interface state. To accomplish this aim, we should first understand the effect of refractive index disorder on light dynamics through the periodic lattices of trivial and non-trivial topologies. The study revealed that the trivial lattice exhibit the onset of *TAL* when the introduced disorder is above a threshold value of 25%. However, the light propagating through its non-trivial counterpart is not effected by the introduced disorder. Thus, it can be easily inferred that the lattices having non-trivial topology are robust and immune to disorder.

To analyze the effect of disorder on light propagation dynamics, a study of the propagation dynamics of a Gaussian beam of wavelength $1.55\mu\text{m}$ is conducted through both the lattices in presence of refractive index disorder. Owing to the low refractive index contrast of the lattices, the propagation dynamics is studied using scalar beam propagation method. The scalar beam propagation method under paraxial approximation is mapped as:

$$i\frac{\partial A}{\partial z} + \frac{1}{2k} \left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} \right) + \frac{k}{n_0} \Delta n(x) A = 0 \quad (5.60)$$

where A represents the amplitude of the wave, k is the wave vector and $\Delta n(x)$ is the variation over a constant background refractive index n_0 . Now, this imposed variation over the constant background refractive index can be mapped by the following equation:

$$\Delta n(x) = \Delta n_p(x) H(x) \quad (5.61)$$

where $\Delta n_p(x)$ is the difference in bi-layer refractive-index and $H(x)$ is a periodic potential governing the lattice. Now due to the introduction of disorder, Eqn. 5.61 is modified to include the term controlling the disorder. Therefore, the modified refractive index profile $\Delta n(x)$ is written as:

$$\Delta n(x) = \Delta n_p(x) H(x) + \Delta n_p(x) C \delta H(x) \quad (5.62)$$

where the first term represents the periodic profile, added to it is the second term denoting the disorderness defined by the random component δ which is controlled by the dimensionless constant C .

The dynamics of light propagation under the effect of refractive-index disorder is studied for both the trivial and the non-trivial lattices. The transverse intensity distribution at the end of 10mm lattice is analyzed to identify the onset of localization. The intensity distributions are ensemble averaged over 500 realizations. Fig. 5.3 (a) through Fig. 5.3 (e) depicts the ensemble averaged transverse intensity profile for a Gaussian beam propagating through the disordered lattice of trivial topology. It can be observed from these plots that as the amount of refractive index disorder is increased, an exponential decay of the central maximum light

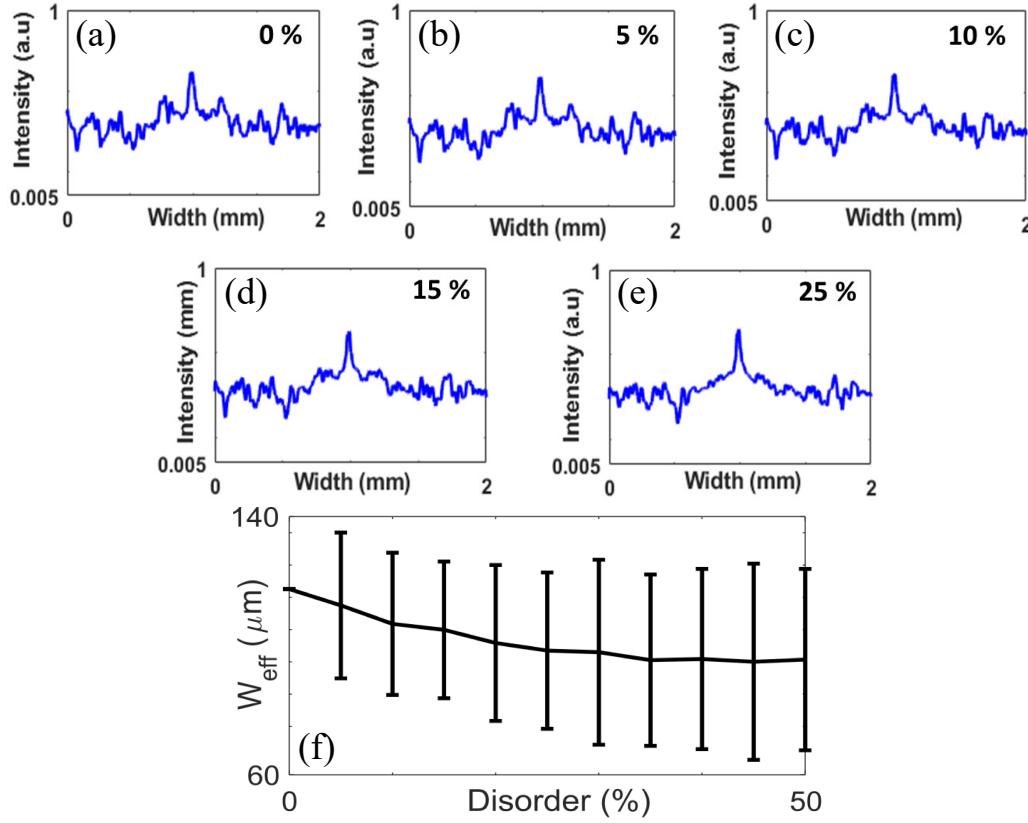


Figure 5.3: The effect on propagation dynamics of light through the topologically trivial lattice in presence of transverse refractive index disorder varying from 0% to 25% is demonstrated here. (a)-(e) Denotes the transverse intensity profile at the end of the 10mm lattice. It can be observed from these plots that as the strength of refractive index disorder is enhanced from 0% to 25%, the transverse spreading of light is arrested and there is an emergence of exponentially decaying tail. (f) The variation of effective width of the transverse intensity profile at the end of the lattice with the amount of refractive index disorder is shown in this plot. It can be observed from this plot that effective width reaches a saturation value as the amount of introduced disorder crosses the threshold amount of 25%.

intensity is observed. As a result, the effective width of the light beam decreases due to transverse trapping. The decreasing value of the effective width with increasing amount of disorder is shown in Fig. 5.3 (f). It can be seen that the beam width gradually decreases and becomes almost constant when the introduced disorder is approximately 25% and more, which indicates the onset of localization.

Similarly, the effect of refractive index disorder on the light dynamics through the non-trivial lattice is also studied as shown in Fig. 5.4. It can be seen that even if the refractive index disorder has been increased upto 25%, there is no signature of the exponential decay of transverse intensity distribution. Hence it is not possible to host the phenomenon of *TAL* in non-trivial lattice with the same amount of disorder introduced to the trivial sub-lattice. However, it has been observed that if we increase the amount of disorder to a much higher value, the transverse spreading of light gets trapped but the non-trivial nature of the lattice do not persists. This fact will be evident in the preceding section.

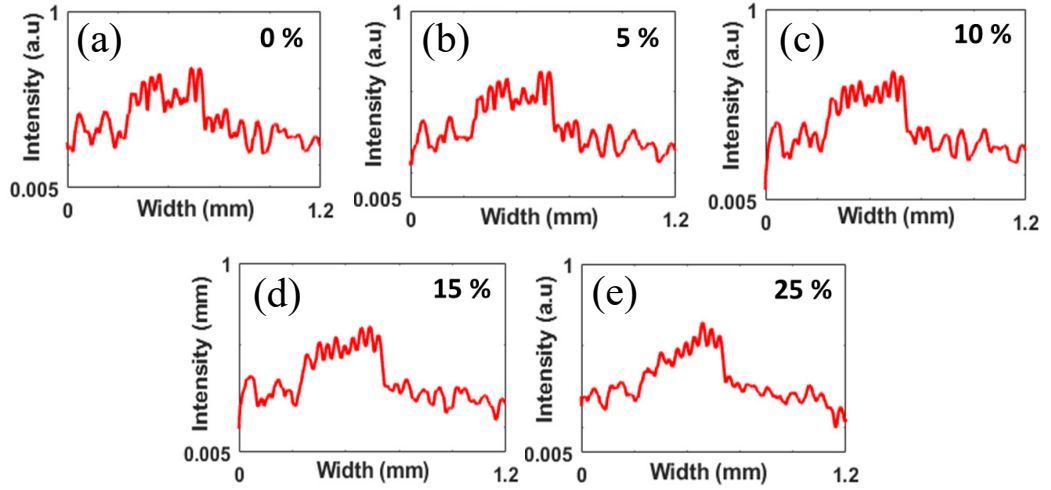


Figure 5.4: The effect on propagation dynamics of light through the topologically non-trivial lattice in presence of transverse refractive index disorder varying from 0% to 25% is demonstrated here. (a)-(e) Denotes the transverse intensity profile at the end of the 10mm lattice. It can be observed from these plots that even if the strength of refractive index disorder is enhanced from 0% to 25%, there is negligible effect on transverse intensity profile. Additionally there is no signature of exponentially decaying tail.

5.3.1 Coexistence of transverse Anderson localized state and topological interface state

To observe the phenomena simultaneously, a 10mm long specialty topological lattice has been considered having thickness on the order of the wavelength of the input Gaussian beam as described in the previous chapter. Deliberate refractive index disorder is introduced along the transverse width of this lattice. Due to the periodic nature of the sub-lattices constituting the topological lattice, it is expected that transverse localized state would be hosted by them. However, there should be a trade-off in the amount of applied disorder, because a very high amount of disorder would hamper the interface-state as the topological nature of the lattices will be changed.

To establish that the *TAL* state can coexist simultaneously with topological interface state, we have launched three Gaussian beams having wavelength of $1.55 \mu\text{m}$ and *FWHM* of $5\mu\text{m}$ simultaneously. The operational regime is deliberately chosen away from the center of the band-gap mentioned in the previous chapter, so that the effect of band-gap guidance can be avoided and the only the effect of disorder can be studied. During this study, one of the beam is launched at the junction of the two sub-lattices and other two are launched at the center of the trivial and the non-trivial sub-lattices. The transverse variation of the ensemble averaged intensity distribution of the input Gaussian beams at the end of the 10mm long specialty topological lattice is shown for different disorder strength in Fig. 5.5.

It can be seen from Fig. 5.5 that, for the ordered lattice (0% disorder) there is no sign of narrowing down of the light beam positioned at the center of the trivial sub-lattice as well as there is no effect on the interface-state. But as the amount of disorder is increased to 10% there is gradual arrest in transverse spreading of the light beam positioned at the center of

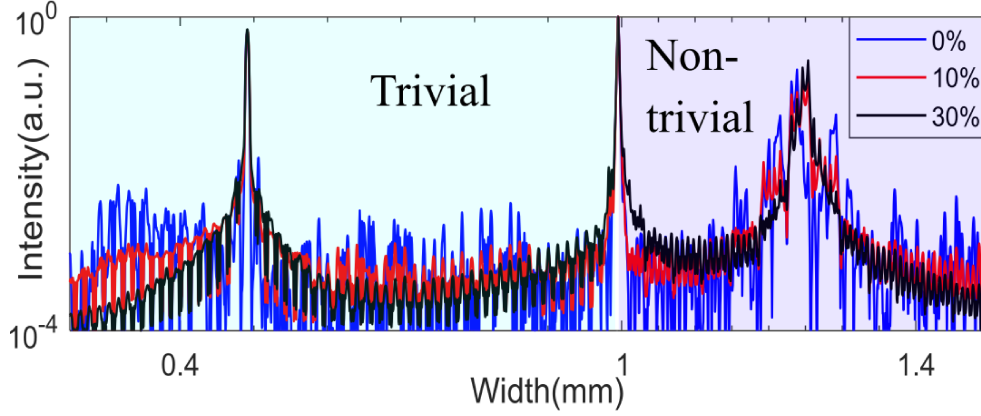


Figure 5.5: This figure represents the effect on light propagation through the specialty topological lattice in presence of deliberate refractive index disorder. The distribution of light intensity at the end of a 10mm long specialty topological lattice in presence of refractive index disorder of different strengths is shown by this plot. The lattice is simultaneously excited by three Gaussian input beams centered at the interface of the two sub-lattices and around the central position of the trivial sub-lattice as well as around the central position of the non-trivial sub-lattice. The beam excited around the central position of the trivial sub-lattice gets localized due to the introduction of optimum disorder whereas there is no effect of disorder on the topological interface state and on the beam excited at the center of the non-trivial sub-lattice.

the trivial sub-lattice, but the interface-state is still not effected. However as the amount of disorder is increased to 30%, it is clearly visible that the light beam launched at the center of the trivial sub-lattice is localized as identified by the exponentially decaying tail of the intensity profile, but the interface-state remains robust to the applied disorder. On the other hand, the Gaussian beam launched around the center of the non-trivial lattice exhibits interesting characteristics. It can be seen that the beam propagation is not similar to that in the trivial sub-lattice. Moreover, the introduced disorder also does not support *TAL*. Therefore it can be concluded that by applying optimum refractive index disorder to the specialty topological lattice, two non-interacting states of light can be simultaneously hosted.

5.3.2 Simultaneous propagation of non-interfering topological interface state and transverse localized state

To establish that the topological interface state and transverse localized state does not interfere with each other in-course of its propagation, the transverse intensity distributions is probed at different lengths of the 10mm long specialty topological lattice as shown in Fig. 5.6. Fig. 5.6 depicts the light propagation dynamics of the Gaussian beam of *FWHM* of $5\mu\text{m}$ at four different wavelengths viz. $1.125\mu\text{m}$, $1.55\mu\text{m}$, $1.8\mu\text{m}$ and $2\mu\text{m}$ indicated by solid, dashed, dash-dotted and dotted lines in Figs. 5.1(d) and (e) respectively. These four wavelengths are chosen so that propagation dynamics can be studied at the center of the operational band-gap, slightly away from the central wavelength explicitly around the C-band, at the edge of the band-gap and outside of the band-gap of the specialty lattice respectively. Figs. 5.6(a), (b) and (c) depicts the propagation dynamics of the Gaussian beam of wavelength $1.125\mu\text{m}$ through the ordered lattice, 10% disordered lattice and 30% disordered lattice respectively. From these three plots it can be inferred that, since the wavelength of operation is at the center of the operational

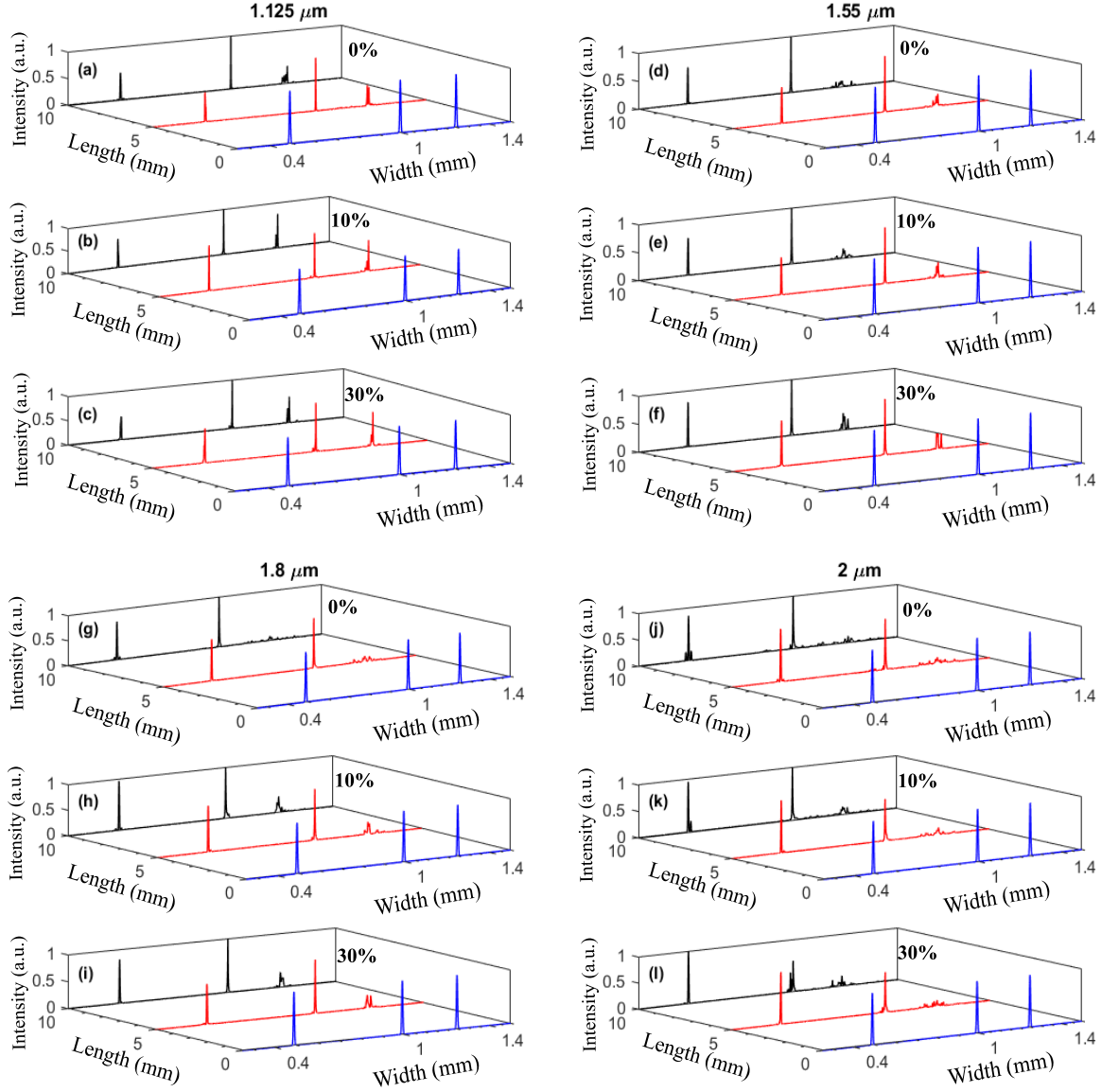


Figure 5.6: The propagation dynamics of the Gaussian beam through the designed specialty lattice is depicted here. The study is conducted at four different wavelengths viz. $1.125\mu\text{m}$, $1.55\mu\text{m}$, $1.8\mu\text{m}$ and $2\mu\text{m}$ which are at the band-center, away from band-center, at the band-interface and out of the band-gap respectively. Moreover, the beam propagation study is conducted for ordered lattice (0% disorder), 10% disordered lattice and 30% disordered lattice. For a specific wavelength of operation as mentioned above, the first row represents the propagation dynamics for the ordered lattice, the second row for 10% disordered lattice and the third row for 30% disordered lattice.

wavelength window, we can observe the effect of transverse localization in the trivial as well as non-trivial sub-lattices, but interestingly the interface-state is not effected. In this scenario, as the operational wavelength is at the center of the band-gap, the non-trivial lattice experience the effect of transverse localization due to the applied disorder. However, as we go slightly away from the central wavelength to $1.55\mu\text{m}$, we can observe that the Gaussian beam launched at the center of the trivial sub-lattice gets localized due to the introduction of the transverse disorder whereas the interface-state as well as the Gaussian beam launched at the center of the non-trivial lattice is not effected by the introduced disorder. The propagation dynamics of the Gaussian beam of $1.55\mu\text{m}$ are shown by Figs. 5.6(d), (e) and (f) for ordered lattice, 10% disordered lattice and 30% disordered lattice respectively. Further, it can be noticed that the guidance of light is very poor through the ordered non-trivial sub-lattice. However, when disorder is introduced, there is certain transverse trapping of light but the state of light formed does not carries the signature of *TAL* state. Now, as the operational wavelength is further shifted to the edge of the band-gap, the confinement of the interface-state is slightly reduced as well as the leaky nature of the Gaussian beam propagating through the trivial lattice also increases. However, as the amount of transverse disorder is increased, the beam propagating through the trivial sub-lattice starts exhibiting the phenomenon of transverse localization. These unique phenomena can be observed from Figs. 5.6 (g), (h) and (i) where the propagation dynamics of the Gaussian beam of $1.8\mu\text{m}$ is demonstrated through ordered lattice, 10% disordered lattice and 30% disordered lattice respectively. For this wavelength also, light does not propagate through the non-trivial sub-lattice and no *TAL* state is formed in it due to the introduction of refractive index disorder. Finally the propagation dynamics of the light operating outside the band-gap is studied. The propagation dynamics of the Gaussian beam of $2\mu\text{m}$ is demonstrated through Figs. 5.6 (j), (k) and (l) for the ordered lattice, 10% disordered lattice and 30% disordered lattice respectively. Figs. 5.6 (j) depicts the propagation dynamics through an ordered lattice operating in the pass-band, where the interface-state is ill confined as well as the beam propagating through the trivial sub-lattice is leaky in nature. However, when 10% disorder is applied, the interface-state remains robust but the beam propagating through the trivial sub-lattice experiences some effect of transverse localization as shown by Fig. 5.6(k). On the other hand, as the amount of disorder is further increased to 30%, the propagation dynamics of the interface-state gets affected. This happens because, as disorder is introduced the topological nature of the sub-lattices get affected, but since the wavelength of operation is chosen outside the band-gap, the topological features of state guided through the interface is affected. However, the beam propagating through the trivial sub-lattice is localized. Therefore, from these observations we can infer that it is possible to host two states of light which do not interfere with each other in-course of propagation through the specialty topological lattice owing to the interplay of deliberate disorder and wavelength of operation. Now, to realize this phenomenon experimentally, we have to launch two light beams simultaneously. One of the light beams should be launched at the junction of the two lattices, to excite the topological interface state. To realize this experimentally we have to bring the laser beam into a convergent lens and place the lattice sample at the focus. The convergent lens must be of very high quality, and it must be held and oriented with micron-scale accuracy. On the other hand, another light beam should be launched at the central region of the trivial lattice to excite the transverse localized state. To observe the effect of transverse localization of light, we need to illuminate the input plane of the lattice. Above the threshold disorder for a given aspect ratio of the lattice, the transverse localized state can be excited independent of input location at the excitation plane. In this way, we could excite the transverse localized state of light. Thus, we need the above two arrangements so that we can excite the topological interface state and transverse localized state of light simultaneously to enhance the light-guiding capacity of the lattice.

5.4 Light dynamics through gain and loss assisted specialty topological lattice

Presence of distributed gain and loss through the coupled wave-guide triggers exotic phenomena like topological insulating phases, complex band-gaps, and modified bulk-boundary correspondence [Song et al., 2020; Silveirinha, 2019; Ao et al., 2020; Longhi, 2019; Rui et al., 2019; Song et al., 2019; Zhu et al., 2020; Wang et al., 2019b; Höckendorf et al., 2019]. Further, the introduction of selective gain and loss to a coupled wave-guide system exhibit novel functionalities like asymmetric mode conversion, enhanced sensing, mode selective optical isolation as well as higher-order mode conversion by exploiting the phenomenon of exceptional point which are not possible in passive systems [Ghosh and Chong, 2016; El-Ganainy et al., 2019; Bhattacharjee et al., 2019; Liu et al., 2019; Laha et al., 2020; Dey et al., 2020; Gandhi et al., 2020; Midya, 2020]. The effect of gain can be realized by doping with rare-earth elements [Huang et al., 2019] whereas the effect of loss is realized by doping with free-carriers. However, most of the reported studies dealing with gain and loss are related to $\mathcal{PT}$ -symmetric systems. The exotic phenomenon is observed when the system transforms from exact- $\mathcal{PT}$ regime to broken- $\mathcal{PT}$ regime [Miri and Alù, 2019; El-Ganainy et al., 2018; Feng et al., 2017; Özdemir et al., 2019]. However, due to the structural uniqueness of our designed topological specialty lattice, the regime of operation is not related to the $\mathcal{PT}$ -symmetric domain. Here, we show that by introducing selective gain, the topological states can be selectively amplified without affecting the extended states of the system and can be used in beam manipulation along with applications where robust amplification of light is required. Usually, the introduction of gain to low-index bulk results in enhanced transverse spreading of light as the strength of complex coupling gets boosted due to the introduced gain. So the photons easily hop to the coupled wave-guides resulting in enhancement of spreading. On the contrary, we discovered that topological interface states do not undergo transverse spreading over a sufficiently long distance because they are resistant to external perturbation.

To study the effect of complex coupling on the propagation dynamics of the topological state, we have simulated the effect of introducing gain/loss to the designed specialty topological lattice. Enough doping is done to overcome a certain threshold level so that the effects of introduced gain and loss can be practically realized in the designed specialty lattice because the topologically protected state is less affected by external perturbations. We have found that a minimum amount of gain/loss $\sim 10^{-4}$ is necessary to perturb the propagating interface-state. The outcomes for the lower values of applied gain/loss are not reported here as the propagating states are robust against them.

5.4.1 Effect of homogeneous gain/loss on light dynamics through the specialty topological lattice

We begin by looking at the simultaneous effect of gain and loss in the specialty topological lattice. A uniform gain of 10^{-4} is applied to the layers having higher refractive index material, and simultaneously a uniform loss of 10^{-4} is applied to lower refractive index bulk. A Gaussian beam is launched at the junction of the two sub-lattices. The obtained results are shown in Fig. 5.7 (a). It can be seen that as the trivial lattice has a comparatively greater amount of high refractive index content, the light leaks into it because the introduction of gain leads to a surge of the transverse spreading of the propagating light beam [Bhattacharjee et al., 2020]. On the contrary, a different light dynamics is observed if a uniform loss of 10^{-4}

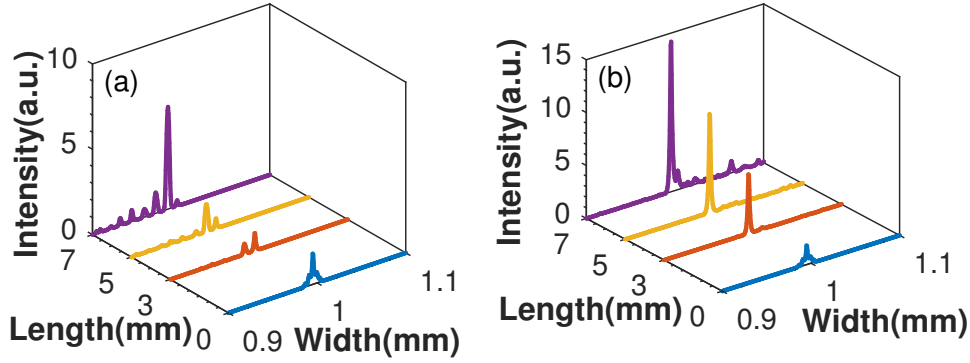


Figure 5.7: The figure demonstrates the propagation dynamics of a Gaussian beam in the presence of simultaneous gain and loss to the specialty lattice. The evolution of light is depicted in (a) with uniform gain of 10^{-4} applied to high refractive index layers and simultaneous loss of 10^{-4} applied to low refractive index bulk, and (b) with uniform loss of 10^{-4} applied to high refractive index layers and simultaneous gain of 10^{-4} applied to low refractive index bulk.

is applied to the layers with higher refractive index material and a uniform gain of 10^{-4} to lower refractive index bulk. The transverse distribution of light intensity across the lattice for this situation is represented by Fig. 5.7 (b). In this scenario, light leaks into the non-trivial sub-lattice, and its amplitude is enhanced. The amplitude of light is increased because gain is introduced to the low index bulk and the lattice have more amount of low-index material compared to high-index material. Additionally, light leaks into the non-trivial sub-lattice because the quantity of lower refractive index bulk is more than the material having a higher refractive index in the non-trivial sub-lattice. Thus we can conclude that, when gain is applied to bulk and simultaneously loss is applied to wave-guides, light leaks into the non-trivial sub-lattice due to enhanced transverse spreading. Similarly, when gain is applied to the wave-guides and at the same time loss is applied to the bulk, light leaks into the trivial sub-lattice.

5.4.2 Effect of selective gain/loss on light dynamics through the specialty topological lattice

Furthermore, the impact of selective gain/loss to the bi-layers is analyzed through beam propagation study. We report four different conditions: the introduction of loss to wave-guides, loss to bulk, gain to wave-guides and gain to bulk. In all the situations, the amount of gain/loss is fixed at 10^{-4} . The beam dynamics for each of the four scenarios are shown in the four plots of Fig. 5.8. We launched a Gaussian beam at the interface for this analysis as well. Fig. 5.8 (a) depicts the beam dynamics under the condition of introducing loss only to wave-guides, Fig. 5.8 (b) depicts the beam dynamics when loss is introduced only to bulk, Fig. 5.8 (c) depicts the condition for introducing gain only to wave-guide and Fig. 5.8 (d) depicts the condition of the introduction of gain only to bulk, respectively. It can be observed from the plots that the introduction of loss to wave-guides has similar effect light dynamics as introduction of gain to bulk. According to reports, adding loss to a wave-guide has a similar effect on light diffusive dynamics as adding gain to bulk, and adding loss to bulk has the similar effect on light diffusive

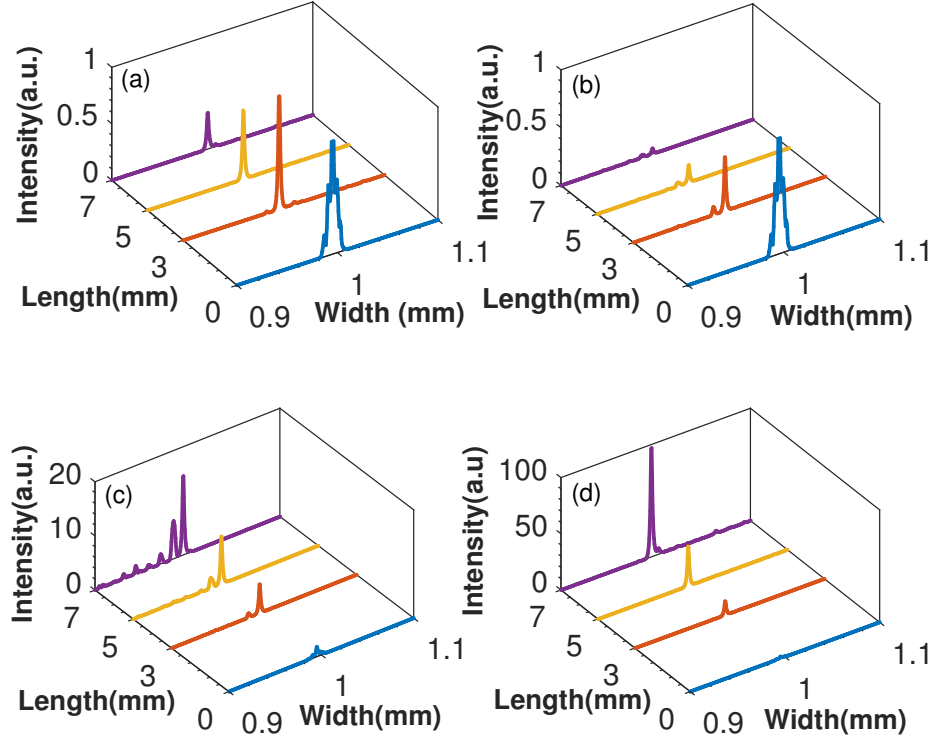


Figure 5.8: The figure demonstrates the light dynamics of the Gaussian beam with selective gain/loss applied to the specialty lattice. The evolution of light has been depicted with (a) uniform loss of 10^{-4} introduced to high refractive index layers, (b) uniform loss of 10^{-4} introduced to low refractive index layers, (c) uniform gain of 10^{-4} applied to high refractive index layers, and (d) uniform gain of 10^{-4} applied to low refractive index layers.

dynamics as adding gain to a wave-guide [Bhattacharjee et al., 2020]. Thus the light dynamics under the reported conditions of the selective introduction of gain/loss is unique.

Additionally, a very exotic phenomenon can be observed from Fig. 5.8(d), which closely relates to a high-intensity topological light state. The intensity of light has increased due to the introduction of uniform gain to bulk, and since both sub-lattices have an equal amount of lower index bulk, the gain is equal in both sub-lattices. So leaking of light in any of the sub-lattices due to the application of gain is prevented. Moreover, we obtain an intense state of light which is highly confined. It can be observed that the enhancement of amplitude is 100 times that of the input beam amplitude. However, careful observation reveals that light leakage is more in the non-trivial sub-lattice than in the trivial sub-lattice at the lattice end when the length of the lattice is more than 7mm. In the current scenario, the strength of complex coupling is more in the non-trivial sub-lattice than in the trivial sub-lattice because the non-trivial sub-lattice contains more low-index bulk where uniform gain is applied. So, an infinitesimally small portion of the light propagating through the interface-state leaks into the non-trivial sub-lattice after 7mm. Further, our preliminary calculations confirm that these infinitesimally small amount of leakage can also be prevented by applying selective loss to few

high-index wave-guides. Currently, research is being conducted in this area in order to propose a design for a robust topological amplifier based on this specialty topological lattice.

5.5 Summary and application

To summarize, we have demonstrated the simultaneous hosting of the *TAL* state and topological interface state in the topological specialty lattice. Due to the interplay of disorder and band-topology, two non-interacting states of light are hosted which propagate through the specialty topological lattice and form two distinct states at the end of the lattice. The proposition is very unique and the lattice can be fabricated using laser writing technique as well as lithography. Additionally, we have studied propagation dynamics of three Gaussian beams simultaneously through the specialty lattice in the presence of refractive index disorder. One of the beams are centered at the junction of the trivial and the non-trivial lattice and the other two are launched around the central region of the trivial and the non-trivial lattice. The study is conducted around four different wavelengths ranging from band-center up to the pass-band. It is observed that when the operating wavelength is not at the center of the operational window, the non-trivial sub-lattice have no effect of transverse refractive index disorder, whereas the trivial sub-lattice exhibits the effect of transverse localization as expected. Moreover, the topological interface-state also remain robust to the applied disorder over the operational window. Therefore by properly exploiting these factors, it is possible to host two non-overlapping state of light simultaneously by the interplay of transverse refractive index disorder in a specialty topological lattice. We believe that the findings would lead a pathway to explore new tool to manipulate light by exploiting the synthesis of both the phenomena of topology and transverse localization simultaneously in the same platform.

On the other hand, the effect of complex coupling on the propagation dynamics of the topological interface state is also investigated. It is found that a minimum of 10^{-4} gain/loss is necessary to be applied to the specialty topological lattice to perturb the topological interface state as these are usually robust to external perturbation. It is observed that, if simultaneous gain and loss is applied, light leaks into one of the sub-lattices. If homogeneous gain is applied to wave-guides along with homogeneous loss to the bulk, light leaks into the trivial sub-lattice. On contrary, if homogeneous gain is applied to bulk along with homogeneous loss to the wave-guides, light leaks into the non-trivial sub-lattice. Moreover, the introduction of selective gain/loss is also investigated. It is found that application of loss to bulk have similar effect as introduction of gain to wave-guides. However, when loss is introduced to bulk the topological interface state gets totally vanished. Additionally, when gain is introduced to wave-guides, the light leaks into the trivial sub-lattice. Further, it is found that application of loss to wave-guides have similar effect as introduction of gain to bulk. However, when loss is introduced to wave-guides the light beam is attenuated but as gain is applied to bulk the light beam is amplified. By utilizing this mechanism, robust amplification of light would be possible.