Declaration

I hereby declare that the work presented in this Thesis titled *Input-Output Dynamic Properties of Complex Networks* submitted to the Indian Institute of Technology Jodhpur in partial fulfilment of the requirements for the award of the degree of *Doctor of Philosophy*, is a bonafide record of the research work carried out under the supervision of *Professor Deepakkumar M. Fulwani* . The contents of this thesis in full or in parts, have not been submitted to, and will not be submitted by me to, any other Institute or University in India or abroad for the award of any degree or diploma.

Ram Niwash Mahia
PG201182006

Certificate

This is to certify that the thesis titled *Input-Output Dynamic Properties of Complex Networks*, submitted by *Ram Niwash Mahia* (PG201182006) to the Indian Institute of Technology Jodhpur for the award of the degree of *Doctor of Philosophy*, is a bonafide record of the research work done by him under my supervision. To the best of my knowledge, the contents of this report, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Deepakkumar M. Fulwani
Ph.D. Thesis Supervisor

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Ram Niwash Mahia
Ph.D. Student

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List of Symbols

Symbol	Description
Δ	Maximum Degree of a Node in the Graph
$D_{g,i}$	Damping Coefficient of the i^{th} Machine in Power Network
$A_{(c,o)}$	$(c,o)^{th}$ Co-Factor Matrix of the Matrix A
$adj(.)$	Adjoint or Adjugate of a Matrix
a_{ij}	ij-th Element Value of Adjacency Matrix A
α_j	j^{th} Element Value of Row Vector α
α	o^{th} Row of the Controllability Matrix (CM)
$Area_i$	Area of the ROA Corresponding i^{th} Driver Node
$\tilde{A}$	Adjacency Matrix of Transformed Network
A	Adjacency Matrix of a Network
B_c	Standard Basis Vector with c^{th} entry 1 and All Other Entries are Zero
β_j	Roots of Numerator Polynomial $C_o adj(sI-A)B_c = 0$
bp_i	i^{th} Bus in Power Network with $i = 1,2,\ldots,h$
$\tilde{B}_i$	Input Matrix of Transformed Network
B	Input Matrix of a Network
cb_i	Normalized Control Backbone of Node i of a Network
C_n	Cycle Networks
C_o	Standard Basis Vector with o^{th} entry 1 and All Other Entries are Zero
CP_{ij}	Conductance of the Transmission Line in Power Network
cw_q	Sum of Product of Weights of Closed Walks of Length q
c	Control Node or Driver Node of a Network (Subscript)
deg_{in}	In-Degree of a Node
deg_{out}	Out-Degree of a Node
D	Diagonal Matrix (or Degree Matrix) of a Network
$\delta_i(>0)$	Radius of the Invariant Ellipsoid $\varepsilon(P_i,\delta_i)$
$\delta(\lambda_i)$	Algebraic Multiplicity of the Eigenvalue λ_i
$\delta_{g,i}$	Rotor Angle of the i^{th} Machine in Power Network
$D(s)$	Denominator Polynomial of TF (Depends on context of use)
$det(.)$	Determinant of a Matrix
$diam$	Diameter of a Network
$d(i,j)$ or d	Shortest Distance Between a Pair of Nodes in Graph G
d_i	Sum of the Weights of Outgoing Edges (or Incoming Edges) of the i^{th} Node
$x(t_f)$	Desired States at Time t_f
$E_{g,i}$	Voltage Modulus of the i^{th} Machine in Power Network
$\varepsilon(P_i,\delta_i)$	Invariant Ellipsoid
$E \in \mathbb{R}^{n\times n}$	Singular Matrix (Depends on context of use)
E	Set of Edges of a Graph
$\gamma_\eta \in \mathbb{R}$	Constant Value
γ_i	i^{th} Element Value of Column Vector γ
gcd	Greatest Common Divisor
$G_{co}(s)$ or $G(s)$	Transfer Function Between the c and o Nodes or a Pair of Nodes
g	DC-Gain of a Network

Symbol	Description
gp_i	i^{th} Generator in Power Network with $i = 1, 2, \ldots, l$
G	Graph
$I_{n\times n}$	Identity Matrix with $n \times n$ Dimension
u	Control Input of a Network
$x(t_0)$	Initial States at Time t_0
$\kappa_i \in \mathbb{R}$	Constant Value
k_{gf}	Gain Factor
$k_g \in \mathbb{R}^+ \cup \{0\}$	Constant Value
K_n	Complete Networks
$K(s)$	Controller Transfer Function
λ_i	Eigenvalues of Matrix A or Matrix L or Roots of Denominator Polynomial
L	Laplacian Matrix of a Network
$[v_l]$	Left Eigenvector Corresponding to $\lambda_1 = 0$ of Laplacian matrix L
l_{ij}	ij-th Element Value of Laplacian Matrix L
$M_{g,i}$	Inertia of the i^{th} Machine in Power Network
$\lvert . \rvert$	Absolute Value
$M(s)$	Closed-Loop Transfer Function
$\mu(\lambda_i)$	Geometric Multiplicity of the Eigenvalue λ_i
μ_q	Constant Values
m_z	Number of Network Zeros
m	Cardinality of Set of Control Inputs U_i
$\mathbb{N}_{>0}$	Set of the Natural Numbers Excluding Zero
$\mathbb{N}$	Set of the Natural Numbers
N_D	Superset of Driver Nodes (or Control Nodes)
N_E	Number of External Dilation Points of a Network
N_I	Number of Internal Dilation Points of a Network
$\mathcal{N}_i$	Neighboring Set of Node i of a Graph
$\lVert . \rVert$	Norm of a Matrix
$NS(s)$	Sensitivity Transfer Function for a Given Input-Output Pair
N_S	Number of Source Nodes of a Network
N_T	Number of Sink Nodes of a Network
$N(s)$	Numerator Polynomial of Transfer Function (TF)
n	Total Number of Nodes of a Network
Ω	Constant Value (≥ 0)
C	Output Matrix of a Network
$\mathbf{1}_{1\times n}$	All 1-Vector
o	Output Node of a Network (Subscript)
$P_{b,i}$	Active Power Demand of the i^{th} Bus in Power Network
$P_{g,i}$	Mechanical Input Power of the i^{th} Machine in Power Network
$\Phi, \Psi > 0$	Positive Constant Values
P_n	Path Networks
$P_p(t)$	Input Power in Power Network
p_q	Total Number of Paths of the Shortest Distance d
$P(\geq 0)$ or $P(> 0)$	Positive Semidefinite or Definite Matrix with Dimension $n \times n$
$Q_{b,i}$	Reactive Power Demand of the i^{th} Bus in Power Network
Q_i	Coefficient Matrices
$\mathbb{R}_{>0}$	Set of the Positive Real Numbers
$\mathbb{R}$	Set of the Real Numbers
$[v_r]$	Right Eigenvector Corresponding to $\lambda_1 = 0$ of Laplacian matrix L
r_{rd}	Relative Degree

Symbol	Description
sat	Vector Valued Standard Saturation Function
$\Gamma(.)$	Standard Gamma Function
S_n	Star Networks
SP_{ij}	Susceptance of the Transmission Line in Power Network
$t_\eta \in \mathbb{R}$	Constant Value
θ_i	Phase Angle of the i^{th} Bus in Power Network
$T_i > 0$	Positive Definite Matrix
$tr(.)$	Trace of a Matrix
$\bar{U}$	Superset of Control Inputs Corresponding to the N_D
$U_i \in \mathbb{R}^{m\times 1}$	Control Input Vector
$V_{b,i}$	Voltage Modulus of the i^{th} Bus in Power Network
$v_i \sim v_j$	Connection Between v_i and v_j
Vol_i	Volume of the ROA
V	Non-Singular Transformation Matrix (Depends on context of use)
V	Set of Nodes of a Graph
W_{G}	Walk Between a Pair of Nodes in Graph $\mathbf{G}$
w_{ij}	Weight of the Edge j of Path i
w	SOP of Weights of Walks Between a Pair of Nodes
W	Edge Weight
x	State Vector of a Network
$y(t) \in \mathbb{R}$	Output of the Network
ζ	Complex Number $\mathbb{C}$
$z_{g,i}$	Transient Reactance of the i^{th} Machine in Power Network
z	State Vector of Transformed Systems

List of Abbreviations

Abbreviation	*Full Form*
ADI	Alternating Direction Implicit
CARE	Continuous-Time Algebraic Riccati Equation
CEE	Control Energy Centrality
CM	Controllability Matrix
CN	Complex Network
DARE	Discrete-Time Algebraic Riccati Equation
DOA	Domain of Attraction
FFT	Ford-Fulkerson Theorem
GCI	Graph Controllability Index
IOT	Internet of Things
LHG	Low and High Gain
LMI	Linear Matrix Inequalities
LQ	Linear Quadratic
LQR	Linear Quadratic Regulator
LTI	Linear Time Invariant
MCC	Minimum Control Counts
MDR	Mason's Direct Rule
MFMCT	Max-Flow Min-Cut Theorem
MIMO	Multiple-Input Multiple-Output
NEOM	New Edge Orientation Method
OM	Observability Matrix
PBH	Popov-Belevitch-Hautus
ROA	Region of Attraction
SCB	Special Coordinate Basis
SISO	Single-Input Single-Output
SOP	Sum of Product
SSE	Steady State Error
TF	Transfer Function