

## Undertaking

I hereby declare that the work presented in this *Thesis* titled *Parameterized Complexity of Tracking Paths* submitted to the Indian Institute of Technology Jodhpur in partial fulfillment of the requirements for the award of the degree of Doctor of Philosophy, is a bonafide record of the research work carried out under the supervision of Dr. Aritra Banik and Prof. Venkatesh Raman. The contents of this thesis in full or in parts, have not been submitted to, and will not be submitted by me to, any other Institute or University in India or abroad for the award of any degree or diploma.



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P16CS002

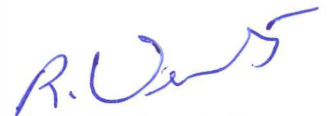


## Certificate

This is to certify that the thesis titled *Parameterized Complexity of Tracking Paths*, submitted by *Pratibha Choudhary (P16CS002)* to the Indian Institute of Technology Jodhpur for the award of the degree of *Doctor of Philosophy*, is a bonafide record of the research work done by her under our supervision. To the best of our knowledge, the contents of this report, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.



*Dr. Aritra Banik*  
*Ph.D. Thesis Supervisor*



*Prof. Venkatesh Raman*  
*Ph.D. Thesis Supervisor*



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*I dedicate this thesis to the loving memories of my Late Grandmother Smt. Rukma Devi, who was always eager to know of my new accomplishments.*

*Pratibha Choudhary*

# List of Publications

Following is list of publications from this thesis.

## Conference Publications:

1. Aritra Banik, Pratibha Choudhary: Fixed-parameter tractable algorithms for tracking set problems. In: Algorithms and Discrete Applied Mathematics - 4th International Conference, CALDAM 2018, Guwahati, India, February 15-17, 2018, Proceedings. pp. 93104 (2018), Springer, Cham.
2. Aritra Banik, Pratibha Choudhary, Daniel Lokshtanov, Venkatesh Raman, Saket Saurabh: A polynomial sized kernel for tracking paths problem. In LATIN 2018: Theoretical Informatics - 13th Latin American Symposium, Buenos Aires, Argentina, April 16-19, 2018, Proceedings, pages 94107, 2018.
3. Pratibha Choudhary: Polynomial time algorithms for tracking path problems. In: Combinatorial Algorithms - 31st International Workshop, IWOCA 2020, Bordeaux, France, June 8-10, 2020, Proceedings. pp. 166179 (2020), Springer, Cham. (Full version can be found at <https://arxiv.org/abs/2002.07799>)
4. Pratibha Choudhary, Venkatesh Raman: Structural Parameterizations of Tracking Paths Problem. In: 21st Italian Conference on Theoretical Computer Science, Ischia, Italy, September 14-16, 2020, volume 2756 of CEUR Workshop Proceedings, pages 1527. CEUR-WS.org, 2020.

## Journal Publications:

1. Aritra Banik, Pratibha Choudhary, Venkatesh Raman, Saket Saurabh: Fixed-parameter tractable algorithms for Tracking Shortest Paths. In: Theoretical Computer Science Journal, 846: 1-13, 2020, Elsevier.
2. Aritra Banik, Pratibha Choudhary, Daniel Lokshtanov, Venkatesh Raman, Saket Saurabh: A polynomial sized kernel for tracking paths problem. In: Algorithmica 82(1) Journal, 4163 (2020), Springer Nature.

## Under review:

1. Pratibha Choudhary, Venkatesh Raman: Improved kernels for tracking path problems. CoRR abs/2001.03161 (2020), <http://arxiv.org/abs/2001.03161>





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