Declaration

I hereby declare that the work presented in this Thesis titled *A game-theoretic perspective of Quantum Information Processing* submitted to the Indian Institute of Technology Jodhpur in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy, is a bonafide record of the research work carried out under the supervision of Dr. Atul Kumar. The contents of this Thesis in full or in parts, have not been submitted to, and will not be submitted by me to, any other Institute or University in India or abroad for the award of any degree or diploma.

Hargeet Kaur
P15CY002

Certificate

This is to certify that the Thesis titled *A game-theoretic perspective of Quantum Information Processing*, submitted by *Hargeet Kaur* (P15CY002) to the Indian Institute of Technology Jodhpur for the award of the degree of *Doctor of Philosophy*, is a bonafide record of the research work done by her under my supervision. To the best of my knowledge, the contents of this report, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Dr. Atul Kumar
Ph.D. Thesis Supervisor

Acknowledgements

All my accomplishments would have been a waste without the grace of **Almighty God**. I am immensely grateful to *Waheguru* for always blessing me with the best and giving me the strength to stand strong despite all obstacles. With this I would like to quote a phrase from the Holy Granth:

ਤੇਰਾ ਕੀਤਾ ਜਾਤੋ ਨਾਹੀ ਮੈਨੋ ਜੋਗੁ ਕੀਤੋਈ ॥
ਮੈ ਨਿਰਗੁਣਿਆਰੇ ਕੋ ਗੁਣੁ ਨਾਹੀ ਆਪੇ ਤਰਸੁ ਪਇਓਈ ॥

– *Sri Guru Granth Sahib Ji*

which means "I have not appreciated what You have done for me. Lord, only You can make me worthy. I am unworthy, and have no worth at all. You have taken pity on me and showered your benedictions upon me." It is His virtues that have made me capable enough to compile this dissertation today.

At this juncture, its a pleasure to thank all the faculty members who played an invaluable role in guiding me through all the years of my initial academic career. I am highly indebted to my supervisor, **Dr. Atul Kumar** for his constant support, attention, and encouragement. He is the best supervisor one could have asked for. Most of this work would have remained incomplete without his critical comments during the lengthy discussion sessions. He has always been patient with me, and given time to pacify my inquisitiveness by answering all my important as well as silly questions, academically and personally. His motivating statements infused with a strong belief in me kept me going even after several failures throughout the journey. Not only this, his constant reminder of self-appreciating every little milestone that one achieves will remain a lesson for life-time.

I also express my deepest gratitude to **Dr. Atul Kumar** and **Dr. Satyabrata Adhikari** for introducing me to the concept of quantum information processing, and **Dr. Bibhas Adhikari** for teaching the basics of matrices and probability theory. Moreover, I extend my special thanks to **Samanwita Pal**, **Dr. Manikandan Paranjothy**, **Dr. Subhashish Banerjee**, **Dr. Ashutosh Kumar Alok**, and **Dr. Ambesh Dixit** for the wonderful courses they taught. The knowledge gained during the courses has been of great help throughout. I also sincerely acknowledge the **Doctoral Committee members** for their positive comments and constructive suggestions during comprehensive examination and State-of-the-Art seminar.

I also wish to thank **IIT Jodhpur** for providing excellent infrastructure and research facility for carrying out uninterrupted research. I am further very thankful to **Dr. Venkata Ramana Badarla** and **Dr. Sandeep Yadav** for allowing me to carry out independent research in their respective laboratories during the difficult phase in the temporary campus of IIT Jodhpur. I also thank **Anuj** and **Vivek** for their kind helping nature during that phase. I would also take the opportunity to thank all the staff members of IIT Jodhpur, especially **Swati Ma'am** and **Poonam Sir** who have always been very supportive, and have helped resolve all official issues that I came across.

This note of thanks will be incomplete without acknowledging my senior **Dr. Parvinder Singh**, my friend-cum-labmate **Jyoti Faujdar**, and **Maitreyee** for their guidance and friendly support. Needless to say, the timely assistance by **Jyoti**, **Abhra**, and **Amrik** during TA duties was another blessing. I am also thankful that I could get the opportunity of being a part of Counseling Service, Students' Gymkhana, and Rotaract Club of IITJ, during different times of my Ph.D. program. The time spent in these clubs helped me sail through the difficult phases in life and evolved me as a much better individual and human being. In addition, it also motivated me to learn new things, interact with various students, make new friends, and grow as a person in non-academic domain. Apart from this, the movies that I got to watch during the Humanities courses taken by **Dr. Vidya Sarveswaran** was another stress-buster in the mundane routine.

Everything goes well only when one remains healthy, not only psychologically but also physically. Thus here, I take great pride to thank my Gymnasium coach **Hem Singh Sir** for working so hard to train me even though I failed to remain punctual and regular for the training sessions. He has been an inspiring personality who filled in the spirit of positive competition within all trainees.

Remembering the past days, I am sure that all would have been impossible without the presence of the most lovely friends and seniors I had. I am very thankful to my friends **Vijendra-aka-Vijju**, **Rahul**, **Kanika**, **Bharat**, **Astha**, **Jyoti**, **Shraddha**, and **Dipti Di** for their care and affection. They have never refused to help whenever I was in need. Though I rarely talked or interacted leisurely with **Astha**, and **Kanika**, they never complained and have always been my well-wishers. I will always cherish the ice-cream and cold-coffee treats by **Rahul** and **Vijju**. Both have been very patient in bearing with me whenever I acted stubborn. I have also been very fortunate to be blessed with wonderful seniors like **Prachi Ma'am**, **Kriti Ma'am**, **Poonam Ma'am**, and **Hiteshi** during my stay at IIT Jodhpur. They have always been there for any kind of suggestion or guidance I needed. My special thanks to **Prachi Ma'am** who has always remained in touch and given me many important tips on life at crucial times.

My heart is filled with joy while I thank **Vijju** for being the most precious person during my Ph.D. and staying by my side through all thick and thins. He has always motivated me to dream big and aspire for the best. If he would not have encouraged me at the right time, I might have given up and lost the courage to continue further. I also extend my heartfelt gratitude to **Vijju's family members** for always being supportive and encouraging. His sisters, **Poonam Di**, **Sonam Di**, and **Supriya Di** have been no less than angels whom I always admire for their ever-loving and caring nature.

Words are not enough to express my gratitude for my father **Mr. Gurpreet Singh Bhatia** and my mother **Mrs. Amarjit Kaur Bhatia** who hold the entire credit for where I stand today. Their blessings and good wishes have enabled me to successfully complete this tough, but important phase. My mother has been a constant guiding light and morale-booster who never let me loose faith in God and myself. The long held friendly discussions with her on phone always filled me with life and positive energy to start afresh. I also seek apology from my sister **Gurbani** and my brother **Hargun** for spending less time with them, and thank them for their love, support, and care.

Hargeet Kaur

List of Figures

Figure	*Title*	*page*
3.1	A comparison of quantum discord for generalized GHZ and $\|\chi\rangle$ states	48
4.1	Maximum payoffs of players vs angle η of the initial state ρ_{new} in the game setting 1	70
4.2	NE payoffs and strategies of players vs angle η of the initial state ρ_{new} when Player 1 applies an amplitude damping noise in the game setting 1	71
4.3	NE payoffs and strategies of players vs angle η of the initial state ρ_{new} when Player 1 applies a phase damping noise in the game setting 1	71
4.4	NE payoffs and strategies of players vs angle η of the initial state ρ_{new} when Player 1 applies a depolarizing noise in the game setting 1	72
4.5	Maximum payoffs of players vs angle η of the initial state ρ_{new} in the game setting 2	72
4.6	NE payoffs and strategies of players vs angle η of the initial state ρ_{new} when Player 1 applies an amplitude damping noise in the game setting 2	73
4.7	NE payoffs and strategies of players vs angle η of the initial state ρ_{new} when Player 1 applies a phase damping noise in the game setting 2	74
4.8	NE payoffs and strategies of players vs angle η of the initial state ρ_{new} when Player 1 applies a depolarizing noise in the game setting 2	74
5.1	Success probability of winning Vaidman's game using GHZ-type states	79
5.2	Success probability of winning Vaidman's game using W-type states	79
5.3	Success probability of winning Vaidman's game using W_n states	80
5.4	Success probability of winning the proposed game where the rule-maker is entangled with the players using a standard W state	82
5.5	Success probability of winning the proposed game where the rule-maker is entangled with the players using a standard GHZ state	83
5.6	Success probability of winning the game with respect to noise parameter ($D_1 = D_2$) using the standard W state	84
5.7	Success probability of winning the game with respect to both the noise parameters ($D_1 \neq D_2$) using the standard W state	84
5.8	Success probability of winning the game with respect to noise parameter ($D_1 = D_2$) using a maximally entangled GHZ state	85
5.9	Success probability of winning the game with respect to both the noise parameters ($D_1 \neq D_2$) using a maximally entangled GHZ state	85
5.10	Success probability of winning the proposed game where the rule-maker is entangled with the players using a four-qubit maximally entangled GHZ state	90
6.1	Relation between the sum of payoffs of players with the concurrence for a combination of BoS and common interest anti-coordination games when different quantum states are shared among the players	98
6.2	Relation between the sum of payoffs of players with the concurrence for a combination of BoS and Chicken games when different quantum states are shared among the players	102
6.3	Relation between the sum of payoffs of players with the concurrence of a general two-qubit Bell state for Bayesian game representation of the tilted Bell-CHSH operator	109

6.4 Relation between the sum of payoffs of players in tilted CHSH game with the concurrence of the Horodecki state for Bayesian game representation of the tilted Bell-CHSH operator 110
6.5 Relation between the sum of payoffs of players in tilted CHSH game with the concurrence of the defined mixed state for Bayesian game representation of the tilted Bell-CHSH operator 111

List of Tables

Table	*Title*	*page*
1.1	The effects of Bob's and Charlie's measurements on Alice's state in a QSS protocol	20
1.2	A payoff matrix for the Prisoners' dilemma game	24
1.3	A payoff matrix for the Battle of sexes game	24
1.4	A payoff matrix for the Stag hunt game	25
1.5	The dependence of winning prospects on inputs received by players in a CHSH game	29
1.6	Payoffs of Alice and Bob in a general game setting where dependence of payoffs on type of players commensurate with the input-output relation in a CHSH game (Here, u_1^A, u_1^B, u_2^A, u_2^B, u_3^A, u_3^B, u_4^A, and u_4^B are non-zero)	30
2.1	Payoffs of Alice in the general PPP game	38
2.2	Payoffs of Eve in the general PPP game	38
2.3	Conditions for (A_i, E_j) to be a Nash equilibrium	40
2.4	Payoffs of Alice in the PPP game with only two weight terms	40
2.5	Payoffs of Eve in the PPP game with only two weight terms	41
2.6	Conditions for (A_i, E_j) to be a NE for eavesdropping excluding DoS attacks	41
2.7	Conditions for (A_i, E_j) to be a Nash equilibrium for an eavesdropper with unlimited power	42
2.8	Payoffs of Alice in the two-way QKD game	43
2.9	Payoffs of Eve in the two-way QKD game	43
2.10	Conditions for (A_i, E_j) to be a Nash equilibrium in the two-way QKD game	44
3.1	Various attacks on PPP using an ω state, two Bell states, or a GHZ state for two bit information transfer using a combination of identity or σ_z operations on the travel photons	57
3.2	Payoffs of the team of a sender and a receiver in the game-theoretic model of PPP for transmission of two bits of message	59
3.3	Payoffs of an eavesdropper in the game-theoretic model of PPP for transmission of two bits of message	59
3.4	Conditions for (A_i, E_j) to be a Nash equilibrium in a three-qubit PP game	60
3.5	Conditions for different Nash equilibria in a three-qubit PP game	61
5.1	Winning probability of the three-qubit proposed game in a noisy environment	83
5.2	The control mode of facilitated information sharing	86
5.3	The message mode of facilitated information sharing	86
5.4	Generalization of Vaidman's Game for Multi-qubit Systems	89
6.1	Payoffs of Alice and Bob when they either play a conflicting interest coordination game similar to the Battle of Sexes game or a common interest anti-coordination game	96
6.2	A payoff matrix for the Chicken game	100
6.3	Payoffs of Alice and Bob when they either play a conflicting interest coordination game similar to the Battle of Sexes game or a conflicting interest anti-coordination game similar to the Chicken game	101
6.4	Payoffs of Alice and Bob in a general game setting where dependence of payoffs on type of players commensurate with the input-output relation in a tilted CHSH game where u_1^A, u_1^B, u_2^A, u_2^B, u_3^A, u_3^B, u_4^A, u_4^B, u_5^A, and u_5^B are non-zero	105

6.5 Payoffs of Alice and Bob in a common interest game setting where dependence of payoffs on type of players commensurate with the input-output relation in the tilted CHSH game 106
6.6 Payoffs of Alice and Bob in a conflicting interest game setting where dependence of payoffs on type of players commensurate with the input-output relation in the tilted CHSH game 107
6.7 Payoffs of Alice and Bob in a conflicting interest game setting involving the Chicken game as an example of an anti-coordination game where dependence of payoffs on type of players commensurate with the input-output relation in the tilted CHSH game 112

List of Symbols

Symbol	*Description*
$\|0\rangle,\|1\rangle,\|+\rangle,\|-\rangle$	Quantum states representing qubits
ρ	Density operator
$\sigma_x,\sigma_y,\sigma_z$	Pauli spin operators
I	Identity operator
$\|\psi\rangle$	An arbitrary quantum state
$\otimes$	Tensor product
H	Hilbert space
Tr	Trace operation
U	Unitary operator
E	Entanglement of formation
γ_s	Schmidt number
C	Concurrence
τ	Three-tangle
σ	Sigma
N	Negativity
\$	Payoff
u_i	Utilities
w	Weights
p_d	Probability of detection of an eavesdropper
$\|\psi^{\pm}\rangle,\|\phi^{\pm}\rangle$	Bell states
$\|\chi\rangle$	Chi states
$\|\omega\rangle$	Omega states
$I(\rho)$	Mutual information
D_A	Quantum discord
D_G	Geometric discord
η^{eff}	Efficiency of a PP protocol
$wk(w)$	Weak measurement operation
$wkrev(w_r)$	Weak measurement reversal operation
B	Bell operator
$\oplus_2$	Addition modulo 2
I_α^β	Bell-type operator

List of Abbreviations

Abbreviation	*Full form*
BB84	A key distribution protocol proposed by Bennett and Brassard in 1984
BoS	Battle-of-the-Sexes
CHSH	Clauser, Horne, Shimony, and Holt
CM	Control mode
CNOT	Control spin flip operation
CPBS	Controlled Polarizing Beam Splitter
DCNOT	Double Control spin flip operation
DoS	Denial-of-Service
EoF	Entanglement of Formation
EPR	Einstein Podolsky and Rosen
GHZ	Greenberger Horne Zeilinger
H	Hadamard operation
I	Mutual information
IR	Intercept and Resend
LM05	A protocol proposed by Lucamarini and Mancini in 2005
LOCC	Local operations and classical communication
MM	Message mode
NE	Nash Equilibrium
NMR	Nuclear Magnetic Resonance
PBS	Polarizing Beam Splitter
Ph.D.	Doctor of Philosophy
POVM	Positive Operator-Valued Measurements
PP	Ping-Pong
PPP	Ping-Pong Protocol
QBER	Quantum Bit Error Rate
QKD	Quantum Key Distribution
QSDC	Quantum Secure Direct Communication
QSS	Quantum Secret Sharing
SWAP	Swap operation