Declaration

I hereby declare that the work presented in this thesis titled *Non-self-adjoint Eigenvalue Problem for Optical Bent Waveguides* submitted to the Indian Institute of Technology Jodhpur in partial fulfillment of the requirements for the award of the degree of Doctor of Philosophy, is a bonafide record of the research work carried out under the supervision of Dr. Kirankumar R. Hiremath. The contents of this thesis in full or in parts, have not been submitted to, and will not be submitted by me to, any other Institute or University in India or abroad for the award of any degree or diploma.

Rakesh Kumar
P18MA001

Certificate

This is to certify that the thesis titled *Non-self-adjoint Eigenvalue Problem for Optical Bent Waveguides*, submitted by *Rakesh Kumar (P18MA001)* to the Indian Institute of Technology Jodhpur for the award of the degree of *Doctor of Philosophy*, is a bonafide record of the research work done by her under my supervision. To the best of my knowledge, the contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Dr. Kirankumar R. Hiremath
Ph.D. Thesis Supervisor

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List of Symbols

SWG	Straight waveguide.
BWG	Bent waveguide.
EVP	Eigenvalue problem.
NSA	Non-self-adjoint.
R	Bent radius.
$\mathbb{R}$	The set of real numbers.
L_R	An operator governing the bent waveguide problem with finite radius R.
L_R^*	An adjoint operator of an operator L_R.
L_∞	An operator corresponds to the equivalent straight waveguide problem.
$\mathbb{V}_{r,R}$	An inner product space in radial coordinates with finite bent radius.
$\mathbb{V}_{x,R}$	An inner product space in Cartesian coordinates with finite bent radius.
$\gamma = \beta - \imath\alpha$	The propagation constant with real phase propagation β and attenuation constant α.
β_s	The propagation constant for the straight waveguide problem.
ϕ	Eigen-function
ϕ_x or ϕ'	Derivative of function ψ w.r.t. x.
$\mathbb{E}(x, y, z, t)$	The electric field vector function of space and time.
$\mathbb{H}(x, y, z, t)$	The magnetic field vector function of space and time.
ϵ and μ	The dielectric permittivity, and magnetic permeability respectively.
k and ω	Wave number, and angular frequency respectively.
n_s, n_f, and n_c	The refractive index of substrate, film, and cladding medium respectively.
TE_0	The fundamental mode for the transverse electric mode. Similarly, for the other mode
$\sigma(L)$	The spectrum of an operator L.
$\sigma_c(L)$	The continuous spectrum of an operator L.
$\rho(L)$	The regular values of an operator L.
R_λ	Resolvent operator with complex λ.
d	Core width
$\|.\|$	The norm of any map.
$\langle ., . \rangle$	An inner product.
$\mathcal{L}^2(\mathbb{R})$	The space of square-integrable functions on $\mathbb{R}$.