

Exceptional points in 1D photonic bandgap waveguide

4.1 Introduction

Photonic Bandgap (*PBG*) waveguides have emerged as a revolutionary concept, redefining the traditional paradigms of light propagation and confinement in the realm of optical waveguiding, unlike conventional waveguides, such as the planar optical waveguides, where light confinement results from Total Internal Reflection (*TIR*), *PBG* waveguides leverage periodic structures to create spectral bands where photon propagation is forbidden. These bands, known as photonic bandgaps, act analogously to electronic bandgaps in semiconductors but for photons. One of the prime advantages of *PBG* waveguides over traditional planar optical waveguides is the ability to confine and manipulate light not just by refraction but by the bandgap itself. This offers unparalleled control over light propagation characteristics, enabling tight bends, compact device footprints, and advanced functionalities. Additionally, because they are less reliant on total internal reflection, *PBG* waveguides can confine light even in low refractive index contrast systems, making them versatile for a variety of materials and platforms.

One of the standout features of *PBG* waveguides in comparison to conventional planar optical waveguides is how they control light. Instead of relying solely on refraction, these waveguides harness the bandgap itself, presenting advantages. This permits more compact designs, tight bends, and the introduction of advanced functionalities. Furthermore, the confinement in *PBG* waveguides is not strictly dependent on total internal reflection, allowing for effective light confinement even in systems with a low refractive index contrast, broadening their adaptability across different materials and platforms. Nevertheless, despite the promising intrinsic features of photonic bandgap waveguides, practical implementation has encountered difficulties arising from obstacles related to lithographic accuracy and scattering losses. Moreover, the waveguiding characteristics have predominantly been crafted using two-dimensional planar photonic bandgap structures. Despite offering a degree of control over light propagation, these structures are somewhat influenced by the lateral configurations. The quest continues for versatile waveguides that are low in loss, yet provide substantial control over various guiding attributes like mode size, effective index tunability, group-velocity dispersion, and more, without compromising on maintaining single-mode operation. The Bragg Reflection Waveguides (*BRW*) was first subjected to a full analysis by Yeh and his research team [Yeh and Yariv, 1976]. It was shown that transverse waveguiding is achieved in the *BRW* not via *TIR*, but rather by dispersed reflection in a periodic cladding. It is worth noting that the guided modes in this structure have effective indices that may be lower than the indices of all materials and, in certain situations, even below unity. This is achievable as long as the transverse propagation vector remains within the Bragg stop band of the cladding. Such a characteristic has unlocked enhanced functionalities in photonic devices, with applications spanning electron accelerators [Mizrahi and Schächter, 2004; Zhang et al., 2005] to phase matching for nonlinear frequency conversion [Helmy and West, 2005]. In addition, *BRWs* have been suggested for applications such as mechanically adjustable air-core filters [Sakurai and Koyama, 2004], polarisation splitters [Simova and Golub, 2003], and the propagation of soliton waves [Wächter et al., 1992]. In the realm of sensing, *PBG* waveguides exhibit heightened sensitivity to changes in refractive indices

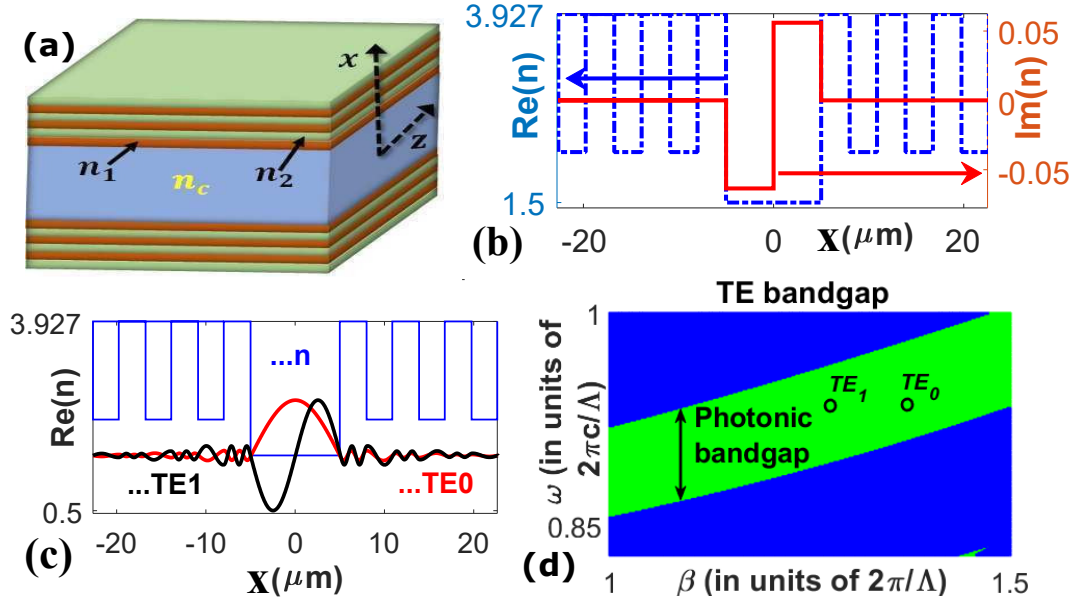


Figure 4.1: (a) Schematic of a planar Bragg reflection waveguide with transverse x -axis (considering propagation along z -axis). (b) Transverse refractive index profile $n(x)$ showing $\text{Re}(n)$ (dotted blue line) and $\text{Im}(n)$ at a specific $\gamma = 0.064$ and $\tau = 0.8894$ (solid red line). (c) Optimized real part of the refractive index $n(x)$, and field amplitude profiles of two quasi-guided TE modes TE_m ($m = 0, 1$). (d) Optimized band diagram of the proposed periodic cladding structure. Here, circular markers indicate the location of the two guided TE-modes in the (ω, β) -plane.

or the presence of specific molecules, a trait that has been harnessed for applications in biosensing and environmental monitoring [Skorobogatiy and Yang, 2009]. Furthermore, the quantum domain has observed the burgeoning potential of these waveguides; their confinement properties are apt for embedding quantum dots, thereby making strides in quantum communication and computing [Lodahl et al., 2004]. Undoubtedly, the multifaceted nature of *PBG* waveguides is steering both research and technological innovations in the photonic arena. It is well known that a non-Hermitian Hamiltonian can describe systems with open boundaries. Recently, gain-loss assisted non-Hermitian systems are of immense interest, both theoretically and experimentally [Miri and Alù, 2019a; Bergholtz et al., 2021; Özdemir et al., 2019]. Exceptional points (*EPs*) are a special type of spectral singularity at which eigenvalues and their corresponding eigenvectors of several modes of the structure coalesce simultaneously, that means multiple different eigenvectors become one, and hence the system Hamiltonian becomes defective [Kato, 1966; Moiseyev, 2011]. A second-order *EP* (*EP2*) can be obtained by tuning the 2D parameter space of the structure [Ghosh and Chong, 2016; Gandhi et al., 2020; Dey et al., 2021b,a]. In this chapter, we have conventionally used the abbreviation *EP* instead of *EP2* to call a second-order *EP*. The presence of an *EP* between two states can be realized by the phenomenon of *ARC* between two respective interacting states with crossing/anticrossing of their real and imaginary parts [Moiseyev, 2011; Heiss, 2000; Heiss and Sannino, 1990; Ghosh and Chong, 2016; Gandhi et al., 2020; Dey et al., 2021b,a]. *EPs* have been observed in various optical systems like waveguides Ghosh and Chong [2016]; Gandhi et al. [2020]; Dey et al. [2021b,a]; Doppler et al. [2016], microcavities [Hodaei et al., 2017; Laha et al., 2020a; Liao et al., 2021], photonic crystals [Ding et al., 2015; Cerjan et al., 2016; Jiang et al., 2020; Mock, 2016] and opto-mechanical systems [Xu et al., 2016] leading to number of novel phenomena such as loss-induced transparency Guo et al. [2009a], topological characteristic [Ding et al., 2016; Ke et al., 2017], slow light [Goldzak et al., 2018], unidirectional light propagations [Yin and Zhang, 2013] with enhanced nonreciprocity [Laha et al., 2020b; Thomas et al., 2016; Choi et al., 2017], non-Hermitian induced flat band [Ge, 2015; Qi et al., 2018], and ultrasensitive detection [Wu et al., 2021; Hodaei et al., 2017; Zeng et al., 2019], lasing control [Peng et al., 2016; Hodaei et al., 2016], and enhanced spontaneous emission [Lin et al., 2016]. A photonic bandgap waveguide consists of a low-index core surrounded by 1D

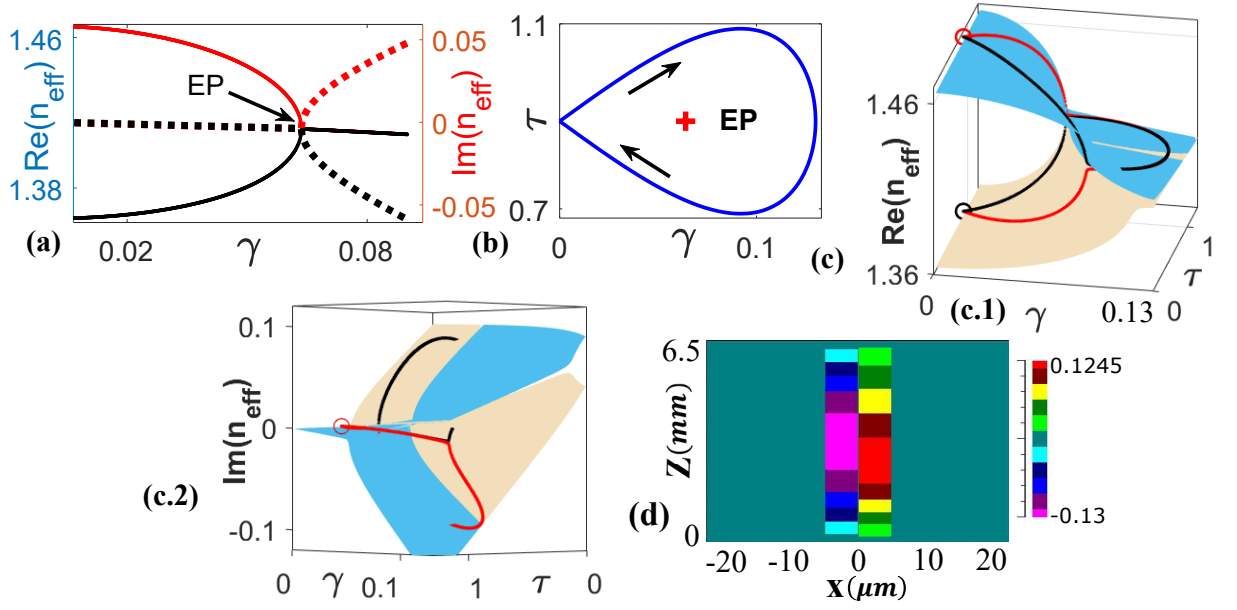


Figure 4.2: (a) Coalescence of $Re(n_{eff})$ (solid red and black line in left y axis) and bifurcation of $Im(n_{eff})$ (dotted red and black line in right y axis). (b) Chosen parameter loop in (γ, τ) -plane to encircle an EP. (c) Structure of the Riemann surfaces associated with (c.1) $Re(n_{eff})$ and (c.2) corresponds to $Im(n_{eff})$ as a function of γ and τ over a specific range. The curves on the surface represent the successive switching of n_{eff} -values due to the parametric encirclement of an EP. (d) length dependent parameter distribution after mapping the loop shown in (b).

periodic cladding on both sides and due to the photonic bandgap effect, we can use it as a low-loss waveguide. The photonic bandgap waveguides provide a full range of functionalities as the light manipulating tools that offer control over guiding properties such as mode size, mode index tunability, group velocity dispersion, etc [West and Helmy, 2006; Pal et al., 2007; Dasgupta et al., 2007]. A quasi-static variation of coupling parameters surrounding an EP permits an interchange between the connecting states, exchanging their initial eigenvalues. In this scenario, one of the eigenvectors gains an additional Berry phase [Dembowski et al., 2004; Menke et al., 2016; Mailybaev et al., 2005]. However, when considering dynamical parametric encirclement around an EP, adiabaticity in system dynamics breaks down, and one of the states behaves nonadiabatically [Gilary et al., 2013; Uzdin and Moiseyev, 2012]. In this context, nonadiabatic corrections become significant due to the dynamical encirclement of an EP. Consequently, time-asymmetric dynamics of the corresponding coupled states are revealed, where one of the eigenstates experiences a lower average loss and dominates at the end of the dynamical parametric variation. Recently, there has been extensive research to explore time-asymmetric light dynamics [Ghosh and Chong, 2016; Gandhi et al., 2020; Dey et al., 2021a; Gilary et al., 2013; Zhang and Chan, 2019]. BRW have proven useful for various optical applications due to the extra degrees of freedom available for parameter selection to control light [West and Helmy, 2006; Pal et al., 2007; Dasgupta et al., 2007]. Such structures can support low-loss propagation of optical modes, even when the refractive indices of the periodic layers exceed that of the core layer. To fully understand the unique characteristics of EPs within photonic bandgap systems and to identify prospective applications effectively, comprehensive investigations on EPs are essential, requiring deliberate adjustments of structural factors within the system. Indeed, EPs have been observed through proper tuning of geometric parameters in photonic crystal slabs [Kamiński et al., 2017; Zhou et al., 2018], and they can exist for any specified wave vector direction. A thorough study of light routing in a photonic bandgap system using a dynamical EP encirclement scheme remains to be completed. Therefore, comprehensive analysis and physical insight into this subject are essential. Exploration of the dynamical encirclement of an EP in a BRW structure supporting two photonic bandgap quasi-guided TE modes could unlock additional degrees of freedom for manipulating light, which is beneficial for integrated signal processing and routing. The term 'quasi-guided

modes' refers to 'weakly-guided' or 'leaky' modes, which are not completely confined within the lower index core region of the designed photonic bandgap waveguide.

This chapter examines the properties of EPs and their roles in light dynamics within photonic bandgap structures. Two particular types of systems serve as focal points for this exploration: a gain-loss assisted one-dimensional (1D) photonic bandgap waveguide that supports two transverse electric (TE) modes and another type of gain-loss assisted 1D photonic bandgap waveguide supporting multiple TE modes [Dey et al., 2022; Dey and Ghosh, 2021]. In both 1D photonic bandgap waveguides, non-Hermiticity is achieved through the spatial transverse distribution of an unbalanced gain-loss profile in the core region. These structures are specifically examined for TE modes, where the out-of-plane magnetic field component (H_z) and in-plane electric field components (E_x and E_y) are the only non-zero field elements. Customized unbalanced gain-loss profiles are introduced based on two tunable parameters to examine the ARC interactions of coupled TE modes. In the first case, The effects of dynamical encirclement schemes around these EPs are also analyzed to explore asymmetric mode conversions between two photonic bandgap quasi-guided modes. Additionally, this structure exhibits a robust EP -aided mode conversion scheme that holds promise against higher fabrication tolerances, rendering photonic bandgap geometries as excellent alternatives for such conversions.

In the other geometry of the photonic bandgap waveguide, four $EP2s$ are identified among the eight bandgap-guided TE modes by deliberately adjusting two independent parameters: the gain coefficient (γ) and the loss-to-gain ratio (τ). Notably, interactions between pairs of modes are found to be mutually exclusive, underlining the independent and non-interacting nature of the identified $EP2s$. Furthermore, the existence of these $EP2s$ enables four unique second-order *flip-of-states* phenomena, revealed by a quasi-static parametric encirclement around each $EP2$ in the (γ, τ) -plane. The findings presented in this chapter not only open up avenues for unconventional applications of EPs but also provide a robust framework for the development of optical mode converters in next-generation photonic integrated systems. Additionally, within the multimodal photonic bandgap-guided platform, there is an exploration of the potential to route various higher-order TE modes, thereby achieving highly robust on-chip higher-order mode conversion devices.

4.2 Light dynamics around an exceptional point in a 1D photonic bandgap waveguide

4.2.1 Design of a 1D Bragg reflection waveguide to host an $EP2$ and encircled parametrically

A 1D photonic bandgap guided planar optical waveguide is constructed, consisting of three bi-layers in both arms, as shown in Fig. 4.1 (a). The core refractive index is $n_{co} = 1.5$, while the refractive indices of the periodic layers are $n_1 = 3.927$ and $n_2 = 2.15$, respectively. The total width of the waveguide measures $45.28, \mu m$, with a core width d_{co} of $10 \mu m$. The widths of the higher and lower periodic index regions are $3 \mu m$ ($= d_1$) and $2.88 \mu m$ ($= d_2$), respectively. The refractive index variation in the cladding region is periodic, adhering to the equation $n(x + \Delta) = n(x)$, where the periodicity of the cladding is defined as $\Delta = d_1 + d_2$. Propagation occurs along the z -direction, with transverse refractive index distribution along the x -axis. The wavelength λ is chosen in such a way that $k = 1$, and the operational length of the waveguide is $6.5 mm$. This waveguide hosts two photonic bandgap-guided TE modes, denoted as TE_m ($m = 0, 1$). A gain-loss profile is incorporated in the core region to introduce non-Hermiticity, where the gain coefficient is γ , and the loss-to-gain ratio is τ . As a result of this gain-loss profile, the spatially patterned imaginary part

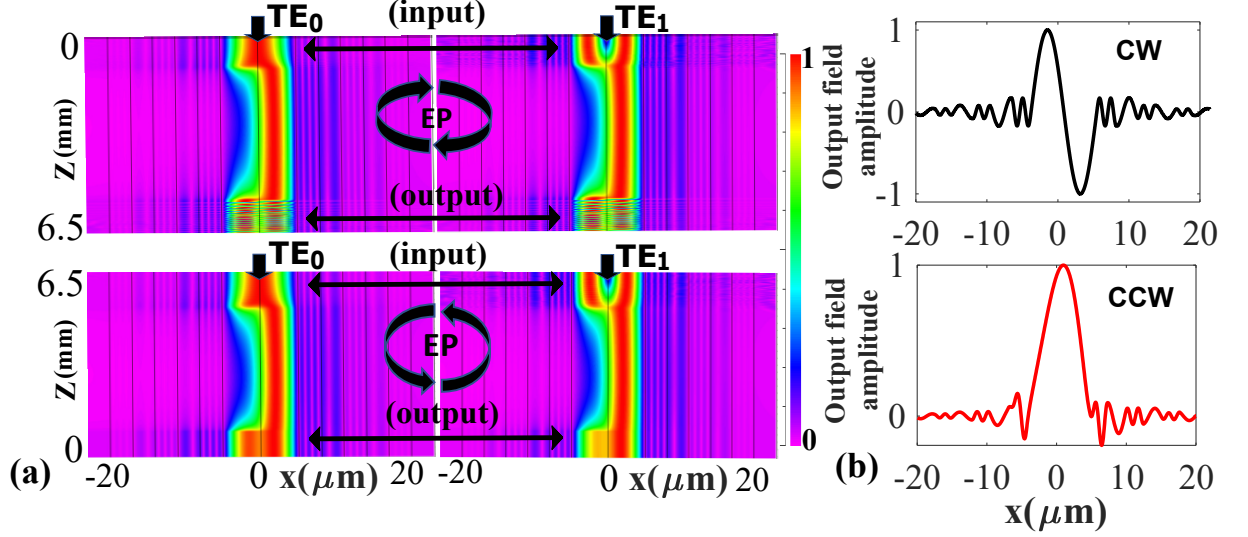


Figure 4.3: (a) Beam propagation simulation results for the propagation of two photonic bandgap guided TE modes (i.e., TE_0 and TE_1) following dynamical parametric encirclement along the loop mentioned in Fig. 4.2 (d). The upper panel of (a) shows the asymmetric conversions $(TE_0, TE_1) \rightarrow TE_1$ for CW dynamical EP encirclement process (i.e., propagation from $z = 0$ to $z = 6.5$ mm). The lower panel indicates the conversion from $(TE_0, TE_1) \rightarrow TE_0$ for CCW dynamical EP encirclement process (i.e., propagation from $z = 6.5$ mm to $z = 0$). (b) The upper panel shows the overall normalized output field amplitude for CW dynamical EP encirclement at $z = L$, related with the conversions of $(TE_0, TE_1) \rightarrow TE_1$ shown in the upper panel of (a). The lower panel indicates the overall normalized output field amplitude at $z = 0$ for CCW dynamical EP encirclement.

of the refractive index can be expressed as follows:

$$n(x) = \begin{cases} n_c + i\gamma, & \text{for } -5\mu\text{m} \leq x \leq 0 \\ n_c - i\tau\gamma, & \text{for } 0 \leq x \leq 5\mu\text{m} \\ n_1, & \text{for } \begin{cases} 5\mu\text{m} \leq |x| \leq 8\mu\text{m} \\ 10.88\mu\text{m} \leq |x| \leq 13.88\mu\text{m} \\ 16.76\mu\text{m} \leq |x| \leq 19.76\mu\text{m} \end{cases} \\ n_2, & \text{for } \begin{cases} 8\mu\text{m} \leq |x| \leq 10.88\mu\text{m} \\ 13.88\mu\text{m} \leq |x| \leq 16.76\mu\text{m} \\ 19.76\mu\text{m} \leq |x| \leq 22.64\mu\text{m} \end{cases} \end{cases} \quad (4.1)$$

The complex refractive index profile for a specific transverse cross-section is depicted in Fig. 4.1 (b). The blue dotted line corresponds to the real refractive index variation, while the solid red line represents the imaginary part of the refractive index variation at specific values of $\gamma = 0.064$ and $\tau = 0.8894$. The supported modes are analyzed using the matrix method [Pal et al., 2007; Dasgupta et al., 2007]. A prototype of the proposed BRW structure can be fabricated through thin-film deposition of Gallium Arsenide ($n_1 = 3.927$) on a thick glass substrate ($n_c = 1.50$), followed by thermal atomic layer deposition of Tantalum Pentoxide ($n_2 = 2.15$) over the Gallium Arsenide ($n_1 = 3.927$) surface [Guo et al., 2009b; Pat et al., 2015; Gougousi and Ye, 2012]. Mode profiles of the two supported TE modes are displayed in Fig. 4.1 (c). The red line corresponds to the lowest-order even mode (i.e., TE_0), and the black line relates to the first higher-order mode (i.e., TE_1). Leakage loss of the transverse magnetic (TM) modes (i.e., TM_0 and TM_1) is estimated to be two orders of magnitude higher than that of the TE modes (i.e., TE_0 and TE_1). Therefore, the presence of TM modes is ignored due to their very high leakage loss. The optimized 1D photonic bandgap for TE modes is illustrated in Fig. 4.1 (d), where circular markers indicate the locations of the two TE modes in the (ω, β) -plane.

The coupling of photonic bandgap quasi-guided TE modes occurs upon introducing non-Hermiticity

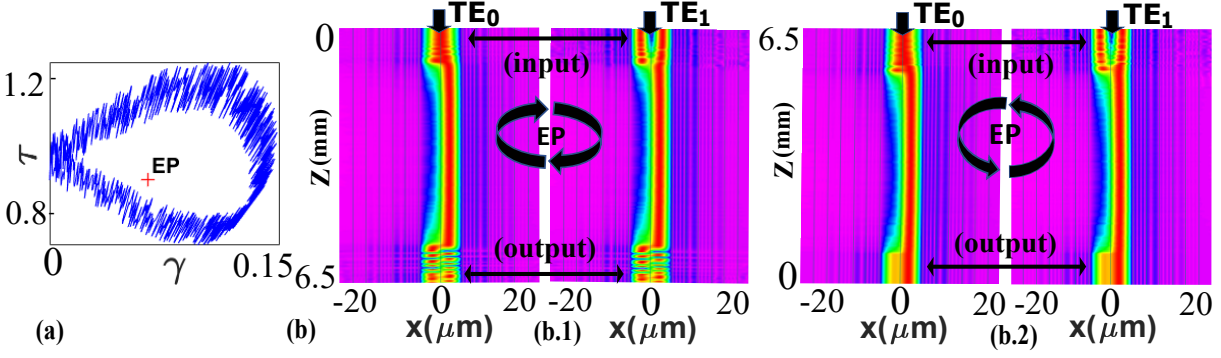


Figure 4.4: (a) The parametric encirclement of an EP is plotted after adding deliberate random fluctuations of 15% on the same parametric loop, which is already shown in Fig. 4.2 (b). (b) Beam propagation simulation results for the propagation of two photonic bandgap guided TE modes (i.e., TE₀ and TE₁) following dynamical parametric encirclement along the loop mentioned in (a). (b.1) shows the asymmetric conversions (TE₀, TE₁) $\rightarrow$ TE₁ for *CW* encirclement process (i.e., propagation from $z = 0$ to $z = 6.5\text{mm}$). (b.2) indicates the conversion from (TE₀, TE₁) $\rightarrow$ TE₀ for *CCW* encirclement process (i.e., propagation from $z = 6.5\text{mm}$ to $z = 0$). Here, we have observed that even after adding 15% of random fluctuations in the parametric loop shown in (a), both the input modes converted into a specific mode for a particular direction of encirclement.

via gain-loss parameters. To encounter an *EP2*, a slow variation of γ within a specified range [0 to 0.09] is considered, and the *ARC* type interaction between the effective mode indices $n_{\text{eff}}(j)[j = 0, 1]$ is studied for two different chosen τ -values [Ghosh and Chong, 2016; Dey et al., 2021a]. At $\tau = 0.8894$, the real parts of the effective mode indices $\text{Re}[n_{\text{eff}}]$ coalesce at $\gamma = 0.064$, accompanied by the simultaneous bifurcation of the corresponding imaginary parts $\text{Im}[n_{\text{eff}}]$, as depicted in Fig. 4.2 (a). This phenomenon establishes the presence of an *EP2* in the (γ, τ) -space at $(\gamma_{EP} = 0.064, \tau_{EP} = 0.8894)$. To further explore the effects of encirclement around the identified *EP2* in the 2D parameter space, a closed parametric variation in the (γ, τ) -plane is applied, followed by solving the coupled equations, as

$$\gamma(\phi) = \gamma_0 \sin \left[\frac{\phi}{2} \right]; \tau(\phi) = \tau_{EP} + a \sin [\phi] \quad (4.2)$$

Here, $\gamma_0 = 0.13(> \gamma_{EP})$ and $a = 0.2(> 0)$, along with ϕ ($0 \leq \phi \leq 2\pi$), serve as the characteristic parameters to construct the encirclement loop. Proper tuning of these parameters enables encirclement of the identified *EP2* in the system parameter space, as depicted in Fig. 4.2 (b). The corresponding Riemann surface for the coupled modes (*TE*₀ and *TE*₁) as a function of the non-Hermiticity parameters γ and τ within the interaction regime is shown in Fig. 4.2 (c). The overall distributions of $\text{Re}[n_{\text{eff}}]$ and $\text{Im}[n_{\text{eff}}]$ are presented in Fig. 4.2 (c.1) and Fig. 4.2 (c.2), respectively, where simultaneous interactions of two $n_{\text{eff}}(j)[j = 0, 1]$ are evident. Hence, it becomes clear that the Riemann surfaces are connected through an *EP2*, as demonstrated in the plots. A subsequent adiabatic mode-switching phenomenon appears on the respective Riemann surfaces due to the encirclement process in the (γ, τ) -plane, enclosing an *EP2*. Observations also reveal that the effective indices of the two modes switch successively following the sequence $n_{\text{eff}}(0) \rightarrow n_{\text{eff}}(1) \rightarrow n_{\text{eff}}(0)$ to complete a loop in the complex n_{eff} -plane, thereby exhibiting second-order branch-point behavior [Ghosh and Chong, 2016] for the corresponding eigenvalues when an *EP2* is *CW* and quasi-statically encircled in the parameter space. Red and black curves indicate the evolutions of the two modes, respectively, from their starting positions. Circular markers of specific colors denote the starting position of the respective trajectory. This form of encirclement process enclosing an *EP2* establishes that the effective indices of the bandgap-guided modes switch successively between them from their respective surfaces.

4.2.2 Dynamical encirclement of an EP toward asymmetric-mode-conversion process

Here, the propagation of the quasi-guided TE modes along the length of the BRW is demonstrated using semi-vector BPM , following the length-dependent refractive index profile as defined in Eq. (4.2). A length-dependent gain-loss distribution must be chosen to encircle an $EP2$ dynamically. This type of dynamical $EP2$ -encirclement process can be achieved by mapping the overall refractive index associated with (γ, τ) -space throughout the length of the BRW , which is equivalent to Eq. (4.2). Such a dynamical EP -encirclement process is realized by considering $\phi = 2\pi z/L$. The following equation can be written as,

$$\gamma(z) = \gamma_0 \sin \left[\frac{\pi z}{L} \right]; \quad \tau(z) = \tau_{EP} + a \sin \left[\frac{2\pi z}{L} \right]. \quad (4.3)$$

In Eq. (4.3), $\phi = 0$ is chosen at $z = 0$ to signify the starting location of the encirclement loop, and $\phi = 2\pi$ is chosen at $z = L$ to mark the end of the encirclement process. Thus, a single complete encirclement, as described by Eq. (4.2), equates to one complete pass of light along the length of the BRW to dynamically enclose an $EP2$ in the system parameters space according to Eq. (4.3). For the CW encirclement process, propagation occurs from $z = 0$ to $z = L$, whereas the CCW encirclement can be realized by reversing the propagation direction from $z = L$ to $z = 0$. This specific choice of parameter space enables the consideration of $\gamma = 0$ at both $z = 0$ and $z = L$, thereby isolating the passive modes and avoiding gain-loss dominating modes at both the input and output ends of the BRW . The overall distribution of the imaginary refractive index profile, corresponding to the encirclement loop in Fig. 4.2 (b), is depicted in Fig. 4.2 (d).

The propagation of TE modes has been studied using the semi-vector BPM via R-Soft[®] simulation. In this study, the two-dimensional semi-vector beam propagation equation is solved, taking into account both the paraxial approximation [Padé order (1,0)] and the adiabatic variation of the imaginary refractive index along the length of the waveguide. It is also established that, during the dynamical $EP2$ -encirclement process, a mode evolving with lower average loss adheres to the adiabatic conversion dictated by the corresponding n_{eff} trajectory. The individual decay rate of a specific TE mode during encirclement around an $EP2$ can be calculated by averaging the losses over the entire path in the complex n_{eff} -plane as [Dey et al., 2021a].

$$\zeta_{average} = \frac{1}{2\pi} \int_0^{2\pi} \text{Im}(n_{eff}) d\theta \quad (4.4)$$

The light dynamics in the photonic bandgap waveguide following the dynamical EP encirclement process [shown in Fig. 4.2 (d)] appear in Fig. 4.3. When light is launched at the $z = 0$ port for executing a CW encirclement scheme, conversion of both TE modes (i.e., TE_0 and TE_1) to TE_1 at $z = L$ occurs, as shown in the upper panel of Fig. 4.3 (a). One adiabatic conversion from TE_0 to TE_1 and one NAT corresponding to TE_1 are observed. The overall normalized output field amplitude appears in the upper panel of Fig. 4.3 (b).

On considering an CCW dynamical EP encirclement scheme by launching light at $z = L$, both modes convert to TE_0 at the end of the process (i.e., at $z = 0$), as depicted in the lower panel of Fig. 4.3 (a). Here, TE_1 undergoes adiabatic conversion, while TE_0 experiences a NAT . The overall normalized output field amplitude is displayed in the lower panel of Fig. 4.3 (b). Adiabaticity is thus not maintained in modal propagation due to the dynamical variation of the non-Hermiticity parameters γ and τ around an $EP2$. This leads to chirality-driven asymmetric mode conversion [Ghosh and Chong, 2016; Gilary et al., 2013; Gandhi et al., 2020], where light converts to a specific bandgap quasi-guided TE mode regardless of the choice of inputs, depending on the direction of propagation. The proposed geometry can be practically realized through standard fabrication techniques. Customized gain-loss profiles can be integrated via controlled variation of the doping concentrations of lossy and gain materials or through an alternative scheme involving controlled non-uniform pumping. Photonic bandgap-guided structures can operate over a wide frequency range, making such EP -aided mode conversion devices advantageous for multi-frequency operation.

Mode conversion efficiencies are also calculated in terms of overlap integrals between the input and

output fields, as

$$C_{i \rightarrow o} = \frac{|\int \psi_o \psi_i dx|^2}{\int |\psi_i|^2 dx \int |\psi_o|^2 dx}; \quad (4.5)$$

where ψ_i represents the input mode profile and ψ_o represents the output mode profile. The maximum efficiency is achieved at an optimized length of 6.5, mm. Maximum conversion efficiency around 92.8% is observed for *CW* encirclement [for conversion $(TE_0, TE_1) \rightarrow TE_1$], while maximum conversion efficiency is 69.45% for *CCW* encirclement [for conversion $(TE_0, TE_1) \rightarrow TE_0$].

The robustness of this *EP*-aided asymmetric mode conversion process against parametric tolerances in state-of-the-art fabrication techniques has also been examined. The dynamical *EP* encirclement process along the loop includes an average of 15% random parametric fluctuations. As depicted in Fig. 4.3, the mode conversion process persists even in the presence of moderately higher parametric tolerances, as shown in Fig. 4.4. Owing to the higher degrees of freedom in choosing structural parameters and demonstrating robust mode conversion phenomena against higher fabrication tolerances, photonic bandgap-guided structures offer new avenues for the development of next-generation optical mode converters.

4.3 Hosting multiple exceptional points in 1D photonic bandgap waveguide for mode engineering

This section introduces a novel prototype of a gain-loss assisted *BRW* that supports nine bandgap-guided *TE* modes. Non-hermiticity in the *BRW* structure is achieved through the spatial variation of a transverse inhomogeneous gain-loss profile. Interactions between specific bandgap-guided modes are regulated by precisely tuning two independent parameters: the gain coefficient γ and the loss-to-gain ratio τ . The system parameter space facilitates the identification of four *EP2s* among the eight bandgap-guided modes by adjusting active parameters γ and τ . Notably, interactions between any pair of modes remain unaffected by the others. This observation indicates that the identified *EP2s* [i.e., $EP2^{(1)}$, $EP2^{(2)}$, $EP2^{(3)}$, $EP2^{(4)}$] are independent and non-interacting. Furthermore, four unique second-order *flip-of-states* phenomena are demonstrated. These phenomena occur during a quasi-static parametric encirclement around each of the four identified *EP2s* in the (γ, τ) -plane.

4.3.1 Design of a 1D Bragg reflection waveguide to host multiple non interacting EP2s and encircled parametrically

The design of the *BRW* structure features a low-index core (refractive index: $n_{co} = 1.458$, width: $d_{co} = 6 \mu m$), which is flanked by identical finite periodic layers, as illustrated in Fig. 4.5 (a). This 1D bandgap-guided structure includes three periodic layers on either side of the core. The spatial widths of the high-index ($n_2 = 3.927$) and low-index ($n_1 = 2.15$) layers are set at $0.1 \mu m$ (d_2) and $0.135 \mu m$ (d_1), respectively. Fig. 4.5 (b) presents a schematic of the complex refractive index profile for specific values $\gamma = 0.035$ and $\tau = 1.5$.

Three periodic layers are selected on each side of the core to ensure maximum confinement of the eight bandgap-guided modes within the core region. This choice of more bilayer number minimizes leakage loss in the system. Bandgap engineering is performed to guide light of wavelength $\lambda = 1.55 \mu m$ using the *BRW*. The spatial periodicity ($\Lambda = d_1 + d_2$) and corresponding refractive indices (n_1 and n_2) of the chosen periodic layers are specifically tailored to create the Bragg stopband. As a result, the waveguide supports nine quasi-guided *TE* modes (i.e., $TE_i; \{i = 0 \dots 8\}$), although only eight of these bandgap-guided modes are shown in Figs. 4.5 (c).

Fig. 4.5 (c.1) illustrates the field confinements of TE_0 (represented by a red curve) and TE_1

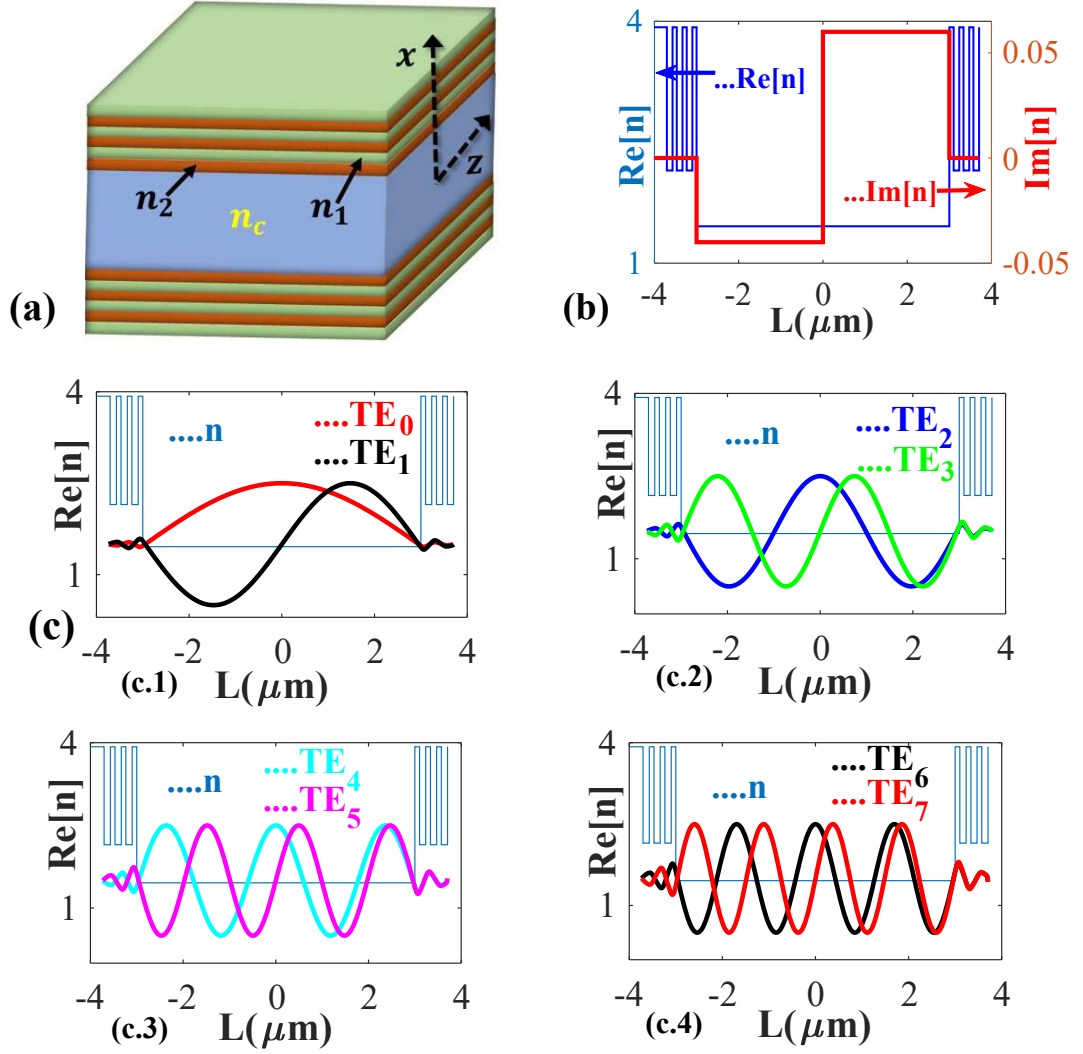


Figure 4.5: (a) Schematic of a planar Bragg reflection waveguide ($x \rightarrow$ transverse axis, $z \rightarrow$ propagation axis). (b) Transverse complex refractive index profile, where $Re[n]$ (blue line) and $Im[n]$ (red line; for $\gamma = 0.035$, and $\tau = 1.5$) represents the real and imaginary parts of the refractive index profiles, respectively. (c) Optimized real part of the refractive index (blue line) of the waveguide and quasi guided TE_i ($i = 0, 1, 2, 3, 4, 5, 6, 7$) polarization modes. (d.1) represents for TE_0 (red curve) and TE_1 (black curve), (d.2) represents for TE_2 (blue curve) and TE_3 (green curve), (d.3) represents for TE_4 (cyan curve) and TE_5 (pink curve) and (d.4) represents for TE_6 (black curve) and TE_7 (red curve).

(represented by a black curve). Similarly, Fig. 4.5 (c.2) displays the field confinements of TE_2 (blue curve) and TE_3 (green curve). The confinements of the pairs of TE_4 and TE_5 , and TE_6 and TE_7 are shown in Fig. 4.5 (c.3) and Fig. 4.5 (c.4), respectively. The presence of TM modes is disregarded due to their relatively higher leakage loss.

In the exploration of bandgap-guided modes, special attention is given to the spatial distribution of a transverse inhomogeneous gain-loss profile that dictates one-to-one coupling interactions among these modes. Through meticulous optimization of operating parameters and unbalanced gain-loss profiles in the core region, four $EP2s$ are identified in the (γ, τ) -plane, as illustrated in Fig. 4.6. The location of $EP2s$ are confirmed through the concept of ARC [Ghosh and Chong, 2016; Dey et al., 2021a], wherein complex effective refractive index values n_{eff} associated with the coupled modes coalesce within the complex n_{eff} -plane.

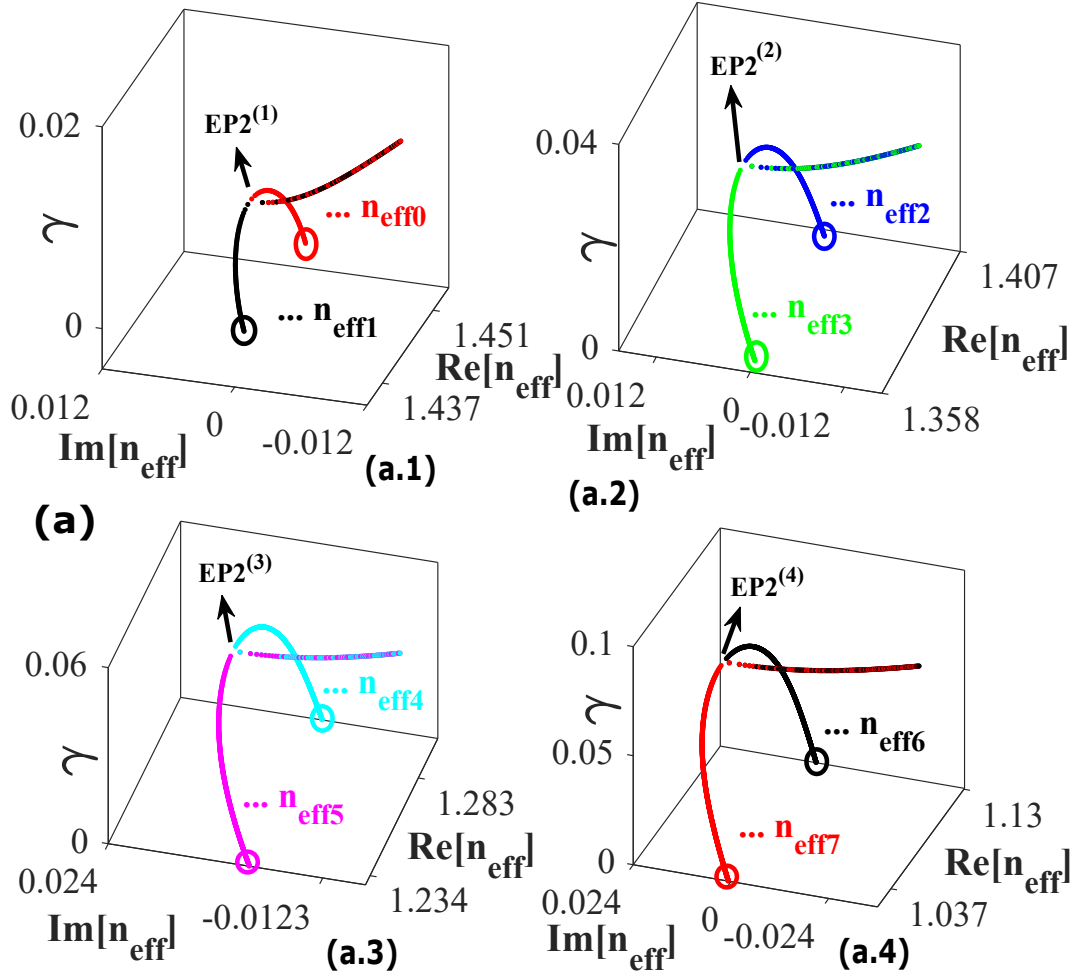


Figure 4.6: (a) Encounter of $EP2s$; (a.1) Dynamics of complex $n_{\text{eff}0}$ (dotted red line) and $n_{\text{eff}1}$ (dotted black line) with respect to γ for chosen $\tau = 1.5$, where the modes coalesce at $EP2^{(1)}$ near $\gamma = 0.0086$. Similarly, (a.2) indicates the dynamics of complex $n_{\text{eff}2}$ (dotted blue line) and $n_{\text{eff}3}$ (dotted green line) for a chosen $\tau = 1.46$, showing coalescence at $EP2^{(2)}$ near $\gamma = 0.0248$. (a.3) Indicates the dynamics of complex $n_{\text{eff}4}$ (dotted cyan line) and $n_{\text{eff}5}$ (dotted pink line) for chosen $\tau = 1.34$, where modes coalesce at $EP2^{(3)}$ near $\gamma = 0.0459$. (a.4) Indicates the dynamics of complex $n_{\text{eff}6}$ (dotted black line) and $n_{\text{eff}7}$ (dotted red line) for a chosen $\tau = 1.25$ showing coalescence at $EP2^{(4)}$ near $\gamma = 0.0698$.

A detailed investigation into the *ARC* phenomena is conducted between two designated modes, focusing on the crossing or anticrossing of their real and imaginary components ($\text{Re}[n_{\text{eff}}]$ and $\text{Im}[n_{\text{eff}}]$). This behavior is examined over two different τ -values and within a specified γ -range. In one case, it is observed that for $\tau = 1.5$, $n_{\text{eff}0}$ and $n_{\text{eff}1}$ coalesce in proximity to $\gamma = 0.0086$, as evident from Fig. 4.6 (a.1). This coalescence numerically locates the position of an $EP2$ at $(\gamma_{\text{EP}} = 0.0086, \tau_{\text{EP}} = 1.5)$, subsequently denoted as $EP2^{(1)}$. In a similar vein, the coalescence of $n_{\text{eff}2}$ and $n_{\text{eff}3}$ near $\gamma = 0.0248$ is discerned, as demonstrated in Fig. 4.6 (a.2). Another $EP2$ is thus numerically pinpointed at $(\gamma_{\text{EP}} = 0.0248, \tau_{\text{EP}} = 1.46)$, and is referred to as $EP2^{(2)}$. Additional analyses identify two more $EP2s$ at coordinates $(\gamma_{\text{EP}} = 0.0459, \tau_{\text{EP}} = 1.34)$ and $(\gamma_{\text{EP}} = 0.0698, \tau_{\text{EP}} = 1.25)$, respectively designated as $EP2^{(3)}$ [Fig. 4.6 (a.3)] and $EP2^{(4)}$ [Fig. 4.6 (a.4)]. By adhering to a rigorous analytical framework, these findings contribute significantly to understanding $EP2s$ in the context of bandgap-guided modes controlled by transverse inhomogeneous gain-loss profiles. The behavior of all identified $EP2s$ exhibits second-order branch point characteristics, as verified by their encirclement in the (γ, τ) -plane. Fig. 4.7 (a) serves as an illustration for exploring the encirclement scheme associated with $EP2s$. In this scenario, a closed elliptical parametric loop in the (γ, τ) -plane is considered, governed by the coupled equations $\gamma(\phi) = \gamma_0 \sin(\phi/2)$ and $\tau(\phi) = \tau_{\text{EP}} + r \sin \phi$, where r is a characteristic

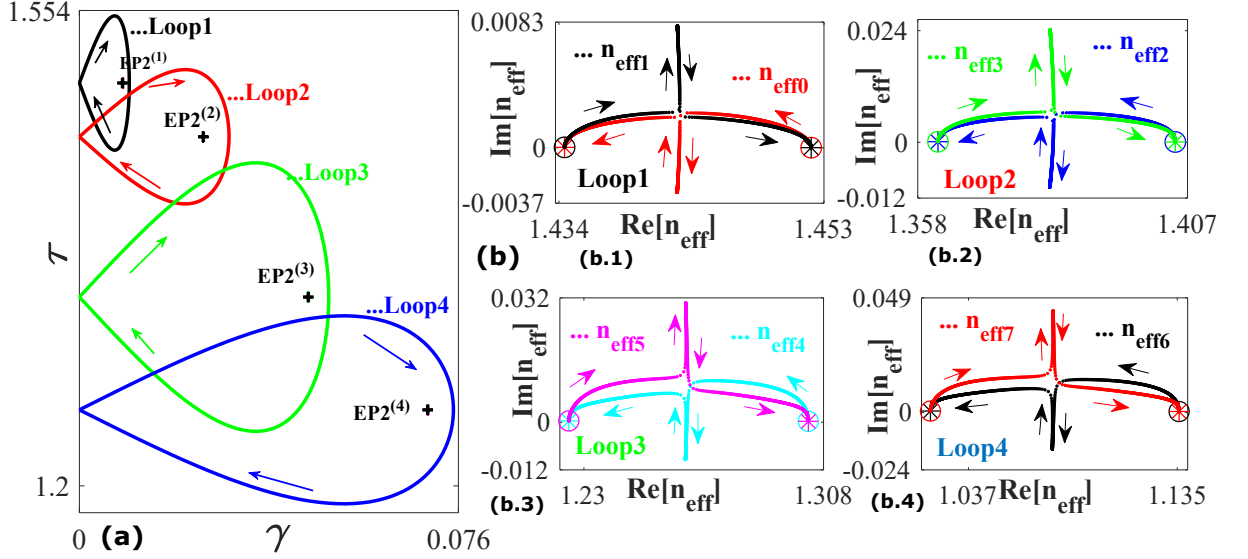


Figure 4.7: (a) Chosen parameter loops in (γ, τ) -plane to encircle individual $EP2$ s, where black loop indicates the encirclement of $EP2^{(1)}$, red loop indicates the encirclement of $EP2^{(2)}$, green loop indicates the encirclement of $EP2^{(3)}$, and blue loop indicates the encirclement of $EP2^{(4)}$. (b) Trajectories of complex $n_{\text{eff}i}$ ($i = 0, 1, 3, 4, 5, 6, 7$) (represented by dotted red, black, blue, green, cyan, pink, black, red) in complex $n_{\text{eff}i}$ -plane following the adiabatic parametric variation in (γ, τ) -plane along (b.1) Loop1, indicates the adiabatic switching between $n_{\text{eff}0}$ and $n_{\text{eff}1}$, unaffected others; (b.2) Loop2, indicates the adiabatic mode exchange between $n_{\text{eff}2}$ and $n_{\text{eff}3}$, un-affecting others; (b.3) Loop3, indicates the adiabatic mode exchange between $n_{\text{eff}4}$ and $n_{\text{eff}5}$, un-affecting others; and (b.4) Loop4, indicates the adiabatic mode exchange between $n_{\text{eff}6}$ and $n_{\text{eff}7}$, un-affecting others. Un-affected trajectories related to respective loops are not shown in the figures. Circular and star markers of the respective colors represent their initial and final position, respectively (i.e., for $\phi = 0$). Arrows of respective colors specify their direction of progressions.

parameter, and ϕ is a tunable angle within $[0, 2\pi]$.

For values of $\gamma_0 > \gamma_{EP}$, the parameter loop confirms the encirclement of the specific $EP2$ in parameter space. This type of encirclement allows the loop to reach $\gamma = 0$ at both its start and end, a feature essential for applications requiring clean passive modes upon completing the encirclement. By judiciously setting the encirclement parameters ($\gamma_0 = 0.01$, $\tau_{EP} = 1.5$, and $r = 0.05$), the loop successfully encloses only $EP2^{(1)}$, as depicted by the black curve in Fig. 4.7 (a). Under such conditions, a single round of parametric encirclement results in the coupled pair $(n_{\text{eff}0}, n_{\text{eff}1})$ of the corresponding TE modes (TE_0 and TE_1) exchanging their initial positions, a phenomenon referred to as *flip-of-states*, and it shown in Fig. 4.7 (b.1).

Subsequent encirclements, specified by tailored parameter sets, can target other $EP2$ s. For example, a particular encirclement delineated by a red curve ($\gamma_0 = 0.03$, $\tau_{EP} = 1.46$, and $r = 0.05$) in Fig. (4.7) (a) focuses on $EP2^{(2)}$. This encirclement leads to a mutual exchange of initial positions between the coupled pair $(n_{\text{eff}2}, n_{\text{eff}3})$, corresponding to TE_2 and TE_3 , as illustrated in Fig. 4.7 (b.2). Analogous *flip-of-states* phenomena occur when enclosing $EP2^{(3)}$. A specific encirclement, marked by a green curve ($\gamma_0 = 0.05$, $\tau_{EP} = 1.34$, and $r = 0.1$) in Fig. (4.7) (a), effects a mutual exchange of initial positions between the paired $(n_{\text{eff}4}, n_{\text{eff}5})$, which correspond to TE_4 and TE_5 . This exchange is captured in Fig. 4.7 (b.3). Similarly, enclosing $EP2^{(4)}$ demonstrates comparable *flip-of-states* phenomena. An encirclement delineated by a blue curve ($\gamma_0 = 0.075$, $\tau_{EP} = 1.25$, and $r = 0.07$) in Fig. 4.7 (a) focuses on $EP2^{(4)}$, inducing a mutual exchange of initial positions for the coupled pair $(n_{\text{eff}6}, n_{\text{eff}7})$, corresponding to TE_6 and TE_7 . This behavior is depicted in Fig. 4.7 (b.4).

Remarkably, the *flip-of-states* phenomena appear to exhibit topological characteristics invariant

to the shape of the parameter space. Mode-swapping within the complex n_{eff} -plane is observed, irrespective of the shape of the encirclement loop. Additionally, the identified $EP2s$ are non-interacting, forming an ‘*Exceptional line*’ in the parameter space. Consequently, mode flipping occurs due to the influence of specific $EP2s$, while the effect of nearby modes remains negligible. The robustness of the EP -aided *flip-of-states* process against parametric tolerances in state-of-the-art fabrication techniques has been examined. In this scenario, deliberate fluctuations of 13% to 15% in each parametric loop [as shown in Fig. 4.7 (a)] do not affect the EP -aided *flip-of-states* processes. This is a significant advantage from a device fabrication perspective.

4.4 Summary

In summary, this chapter is structured into two main sections. The first part delves into a 1D photonic bandgap waveguide assisted by gain-loss mechanisms. This waveguide supports two quasi-guided TE modes that are analytically connected through a $EP2$. Adjusting the transverse gain-loss distribution in the waveguide’s core region allows interactions between these TE modes to be meticulously controlled. The characteristics of these bandgap-guided modes around an $EP2$ are investigated, revealing that the coupled modes undergo an adiabatic interchange of identities when enclosed within a custom-defined, gain-loss two-dimensional parameter space. Further exploration shows that a dynamic encirclement of an $EP2$, achieved through quasi-static variation of system parameters along the waveguide’s propagation length, imparts chiral properties to the device. As a result, different dominant TE modes are delivered at the output, depending on the direction of propagation. This represents an efficient scheme for asymmetric mode conversion and exhibits robustness against high fabrication tolerances. The second part of the chapter introduces a 1D Bragg reflection waveguide characterized by a transversely inhomogeneous gain-loss profile. This structure hosts four non-interacting $EP2s$ and supports eight consecutive bandgap-guided TE modes, which are analytically connected by these $EP2s$ through one-to-one coupling restrictions. When these $EP2s$ are encircled in their respective parameter spaces, a consistent phenomenon of *flip-of-states* becomes apparent. This is exemplified through the second-order branch point behavior of the identified $EP2s$, manifesting as successive mode-switching during quasi-static parametric encirclements around each $EP2$. Both proposed photonic bandgap structures offer extra degrees of freedom in parameter selection. This flexibility is instrumental in investigating the multifaceted properties of $EP2s$ and their corresponding effects on transmission and reflection spectra. Additionally, these structures are capable of operating within a range of specified frequency range, further facilitating the design and tuning of the bandgap. Such capabilities open up novel avenues for applications in next-generation all-optical communication and computing technologies, particularly in achieving chip-scale device footprints and enhanced prospects for selective mode conversion.