Declaration

I hereby declare that the work presented in this thesis entitled *A study of nonclassicality in (Open) Quantum Systems* submitted to the Indian Institute of Technology Jodhpur in partial fulfillment of the requirements for the award of the degree of Doctor of Philosophy, is a bonafide record of the research work carried out under the supervision of Dr. Subhashish Banerjee. The contents of this thesis in full or in parts, have not been submitted to, and will not be submitted by me to, any other Institute or University in India or abroad for the award of any degree or diploma.

Javid Ahmad Naikoo
P14PH003

Certificate

This is to certify that the thesis titled *A study of nonclassicality in (Open) Quantum Systems*, submitted by *Javid Ahmad Naikoo (P14PH003)* to the Indian Institute of Technology Jodhpur for the award of the degree of *Doctor of Philosophy*, is a bonafide record of the research work done by him under my supervision. To the best of my knowledge, the contents of this report, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Dr. Subhashish Banerjee
Ph.D.Thesis Supervisor

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Javid Naikoo.

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List of articles

This thesis is based on the following articles:

1. **Javid Naikoo**, Ashutosh Kumar Alok, Subhashish Banerjee, S. Uma Sankar, "Leggett-Garg inequality in the context of three flavour neutrino oscillation", Physical Review D **99**, 095001 (2019).

2. **Javid Naikoo**, Ashutosh Kumar Alok, Subhashish Banerjee, S. Uma Sankar, Giacomo Guarnieri, Christiane Schultze, Beatrix C. Hiesmayr, " *A quantum information theoretic quantity sensitive to the neutrino mass-hierarchy*", Nuclear Physics B **951**, 114872 (2020).

3. **Javid Naikoo**, Ashutosh Kumar Alok, Subhashish Banerjee, "*Study of temporal quantum correlations in decohering B and K meson systems*", Physical Review D **97**, 053008 (2018).

4. **Javid Naikoo**, Subhashish Banerjee, " *Entropic Leggett-Garg inequality in neutrinos and B (K) meson systems*", European Physical Journal C, **78** 602 (2018).

5. **Javid Naikoo**, Swati Kumari, Subhashish Banerjee, A. K. Pan, "*Probing inequivalent forms of Legget-Garg inequality in subatomic systems*", Journal of Physics G: Nuclear and Particle Physics, **47**, 095004 (2020).

6. **Javid Naikoo**, Subhashish Banerjee, R. Srikanth , " *Leggett-Garg inequality violation under non-Markovian noise*", arXiv:1806.00537.

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8. **Javid Naikoo**, Subhashish Banerjee, Kishore Thapliyal, Anirban Pathak, "*Quantum Zeno effect and nonclassicality in a PT symmetric system of coupled cavities*", Physical Review A **99**, 023820 (2019).

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10. **Javid Naikoo**, Subhashish Banerjee, Anirban Pathak, "*Interplay between nonclassicality and PT symmetry in an effective two level system with open system effects*", Physical Review A **100**, 023836 (2019).

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12. **Javid Naikoo**, Subhashish Banerjee, Arun M. Jayannavar, ''*Violation of Leggett-Garg type inequalities in a driven two level atom interacting with a squeezed thermal reservoir*", Physical Review A **100**, 062132 (2019).

13. **Javid Naikoo**, Subhashish Banerjee, "*Quantumness of channels*", arXiv:1911.07677 [To appear in Quantum Information Processing].

14. **Javid Naikoo**, Swati Kumari, Subhashish Banerjee, A. K. Pan, "*Maximal coherent behavior about exceptional points in a $\mathcal{PT}$ symmetric qubit*", arXiv:1912.12030.

15. **Javid Naikoo**, Subhashish Banerjee, R. Srikanth, A. K. Pan, *Probing non-Markovianity via generalized measurements*, Manuscript under preparation.

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1. Khushboo Dixit, **Javid Naikoo**, Subhashish Banerjee, Ashutosh Kumar Alok, "*Quantum correlations and the neutrino mass degeneracy problem*", European Physical Journal C, **78** 914 (2018).

2. Khushboo Dixit, **Javid Naikoo**, Banibrata Mukhopadhyay, Subhashish Banerjee, "Quantum correlations in neutrino oscillations in curved spacetime", Physical Review D **100**, 055021 (2019).

3. Khushboo Dixit, **Javid Naikoo**, Subhashish Banerjee, Ashutosh Kumar Alok, " *Study of coherence and mixedness in meson and neutrino systems*", European Physical Journal C **79** 96 (2019).

Acronyms

AD	Amplitude Damping
CHLGI	Clauser-Horne form of Leggett-Garg Inequality
CPTP	Completely Positive and Trace Preserving
$\mathcal{CP}$	Charge Parity
ELGI	Entropic Leggett-Garg Inequality
EP	Exceptional Point
GAD	Generalized Amplitude Damping
GDC	Generalized Depolarizing Channel
IH	Inverted Hierarchy
LGI	Leggett-Garg Inequality
LGtI	Leggett-Garg type Inequality
NCP	Not Completely Positive
NH	Normal Hierarchy
NMD	Non-Markovian Dephasing
PD	Phase Damping
POVM	Positive Operator Valued Measure
$\mathcal{PT}$	Parity-Time
QFI	Quantum Fisher Information
RTN	Random Telegraph Noise
SGAD	Squeezed Generalized Amplitude Damping
TTC	Two Time Correlation
WLGI	Wigner form of Leggett-Garg Inequality