Abstract

In nature, the number of individuals of a species or a community of species varies over time within a region or territory. To address challenges in studying living systems due to these variations, mathematical modeling plays a crucial role in the development of an integrative point of view. Mathematical modeling of the population deals with the growth of the species and intrinsic interactions between each organism and its environment. The population modeling was pioneered by Verhulst in the 19th century with the introduction of the Logistic model. Among the several single-species models, one of the crucial population models is the Ricker population model, which was proposed to mathematically represent the stock and fisheries. The Ricker map and its various modified forms have been reviewed by several researchers [Ricker, 1954; Elaydi and Sacker, 2010; Liz, 2018; Rocha and Taha, 2020]. When the growth function of the Ricker map is defined by the Holling type II functional response, then the resulting map is a Homographic Ricker map. The nonlinear dynamics and bifurcation structure of the Homographic map have been discussed by L. Rocha in [Rocha et al., 2020].

The aim of the thesis is primarily to investigate the diverse dynamical properties of the q-deformed Ricker map, the q-deformed Homographic map, the 2D Homographic Ricker map, and the delayed 2D Homographic Ricker map. This study mainly focuses on the various dynamical aspects of these models, which involves the analysis of their nonlinear dynamics, singularities, intersections of different fold and flip bifurcation curves using bifurcation theory, and the exploration of the transition from periodic to chaotic attractors.

In the first part of the thesis, we apply a deformation scheme [Jaganathan and Sinha, 2005; Tsallis, 1988] to the classical Ricker map and obtain a q deformed Ricker map, namely the q-Ricker map. We show that the q-Ricker map proclaims many exciting phenomena that are remarkable in one-dimensional dynamical systems, such as the presence of coexisting attractors, physically non-observable chaos, hydra paradox, bubbling effect, and extinction. We prove that the intersection of the fold and flip bifurcation of the curve gives a singular point of codimension greater than two, and that singular point merges with its associated cusp point. Finally, we show that a certain amount of deformation in the system can keep it in equilibrium; however, excessive deformation causes extinction [Aishwaraya et al., 2022].

Next, we discuss the analytical study of the q-deformed Homographic map (q-Homographic map).The notions of false derivative and the generalized Lambert W function of the rational type are useful in estimating the upper bound on the number of fixed points of q-Homographic map. Further, we explore the process of chaotification of the q-deformed map to enhance its complexity which involves incorporating the residue obtained from multiple scaling of the map's value for the subsequent generation through the utilization of the multiple remainder operator [Moysis et al., 2023]. After the chaotification, the q-Homographic map shows high complexity and the presence of robust chaos, which has been theoretically and graphically analyzed using various dynamical techniques. In addition, we use the feedback control technique [Din, 2017] to control the period-doubling bifurcation and chaos in the q-Homographic map.

In the second part of the thesis, we apply the Holling type - II functional response as a growth function in the classical two-dimensional Ricker map and propose a discrete-time competition model, namely the two-dimensional (2D) Homographic Ricker map. We discuss the boundedness of the solutions and the uniqueness of the coexisting fixed point of the proposed map. With the help of critical curves and singular points, we explain the geometry of the map and prove that all the

points in the domain of the 2D Homographic Ricker map are either regular, fold, or cusp in nature. Furthermore, we use the centre manifold theory to explain the local stability of the fixed points of the proposed map. Using bifurcation theory, we derive some conditions under which the map exhibits the flip bifurcation [Aishwaraya and Chandramouli, 2023].

We further introduce a delayed 2D Homographic Ricker map by incorporating the delay terms in survival functions and small leak terms in the competing populations. We analyze the persistence, boundedness, invariance, and asymptotic behavior of the proposed map. Additionally, numerical simulations are employed to elucidate the stability and bifurcation analysis of the competing population.

In the final part of the thesis, we study the combinatorial tools, namely the Hofbauer tower and the kneading map for a class of symmetric bimodal maps. We discuss the construction, various properties, and geometrical interpretation of these tools. Further, with the help of the Hofbauer tower, we define the cutting times associated with the bimodal map and propose an algorithm to compute the cutting times. Finally, we describe the splitting and co-splitting of the kneading invariants using the cutting and co-cutting times, respectively.