Certificate

This is to certify that the thesis titled *An investigation on the nonclassical and quantum phase properties of a family of engineered quantum states*, submitted by *Priya Malpani (P14EN001)* to the Indian Institute of Technology Jodhpur for the award of the degree of *Doctor of Philosophy*, is a bonafide record of the research work done by her under my supervision. To the best of my knowledge, the contents of this report, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.



Dr. V. Narayanan
Ph.D. Thesis Supervisor

Acronyms

BS	Binomial state
DFS	Displaced Fock state
ECS	Even coherent state
HOA	Higher-order antibunching
HOSPS	Higher-order sub-Poissionian photon statistics
HOS	Higher-order squeezing
KS	Kerr state
LE	Linear entropy
PADFS	Photon added displaced Fock state
PSDFS	Photon subtracted displaced Fock state
PABS	Photon added binomial state
PAECS	Photon added even coherent state
PAKS	Photon added Kerr state
PASDFS	Photon added then subtracted displaced Fock state
VFBS	Vacuum filtered binomial state
VFECS	Vacuum filtered even coherent state
VFKS	Vacuum filtered Kerr state

Declaration

I hereby declare that the work presented in this thesis entitled *An investigation on the nonclassical and quantum phase properties of a family of engineered quantum states* submitted to the Indian Institute of Technology Jodhpur in partial fulfillment of the requirements for the award of the degree of Doctor of Philosophy, is a bonafide record of the research work carried out under the supervision of Dr. V. Narayanan. The contents of this thesis in full or in parts, have not been submitted to, and will not be submitted by me to, any other Institute or University in India or abroad for the award of any degree or diploma.

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