Abstract

Optics has emerged as a significant contributor to technological advancements, especially in optical communication and optical signal processing. Nowadays, with optical waveguides, it is possible to have all the facilities for communication and transmitting high-speed data. Due to its importance, researchers worldwide emphasize exploring optical waveguides and their different aspects, including design, properties, applications, and integration with other technologies.

The most prominent waveguides are straight waveguides and bent waveguides. These waveguides are well-studied experimentally, numerically, and semi-analytically. The mathematical aspects of the optical straight waveguides are explored very well. In this thesis, we presented the mathematical aspects of the dielectric optical bent waveguides. The analysis was done by constructing an eigenvalue problem governing the bent waveguide model using Maxwell's equations in the appropriate space setting. We restrict our-self to the $1-D$ straight and bent waveguide model. The main difficulty in studying the mathematical aspects of the bent waveguide model was the non-self-adjointness of the bent waveguide problem. Unlike Sturm-Liouville's theory for self-adjoint eigenvalue problems, there is no general theory for non-self-adjoint eigenvalue problems. It makes extracting information directly about the model associated with these problems challenging without conducting a separate analysis.

In the Chap.1, we discussed the basics to understand how the optical waves propagate in the planner straight and bent waveguides. The rigorous mathematical model of the straight and bent slab waveguides with constant step-index profiles is discussed. The main objective of the model set-up is to understand the optical wave propagation analytically in the $1-D$ model. The literature survey for the optical waveguides is also a part of this chapter, where one can find the work based on experimental, numerical, and semi-analytic studies.

For the mathematical study of the bent waveguide model, in Chap.2 we discussed some primary results on the spectral theory of self-adjoint and non-self-adjoint operators. We conducted a literature survey focused on the spectral theory of various non-self-adjoint operators. This survey aimed to provide insight into methodologies for analyzing different non-self-adjoint problems.

The analytic study of the straight waveguides showed that the corresponding eigenvalues problem is self-adjoint. It has real eigenvalues. Also, the eigenfunctions corresponding to distinct eigenvalues are orthogonal. The analytical study of the bent waveguide was still missing due to the non-self-adjointness of the corresponding eigenvalue problem. Our study has addressed this gap, a part of Chap. 3. The analytic work presented in this chapter showed the non-self-adjointness of an eigenvalue problem based on an operator theoretic setting. Here, the non-self-adjointness of the bent waveguides problem is discussed by finding the adjoint operator of the problem. The non-self-adjoint problem has non-real eigenvalues, which indicate the lossy nature of the bent waveguide modes. This problem contains a bent radius parameter. The other studies show that when this parameter is large, this problem transforms into a straight waveguide problem. In terms of the underlying mathematics, we proved this using a transformation, showing that a non-self-adjoint problem transforms into a self-adjoint problem. Moreover, the non-real eigenvalues of the bent waveguide problem change into real eigenvalues of the corresponding equivalent straight waveguide problem.

An explicit relation between the real and imaginary parts of the non-real-valued propagation constants is constructed on a detailed analysis. Based on this relation in Chap. 4, the boundedness of both real and imaginary parts of the propagation constants is proved, meaning they are confined within certain region in the complex plane. Furthermore, a self-adjoint problem has

orthogonal eigenfunctions corresponding to distinct eigenvalues. For a fixed bent radius, a $1-D$ semi-analytic study shows that the bent waveguide eigenvalue problem has orthogonal eigenfunctions corresponding to distinct eigenvalues. To prove this analytically, we use the adjoint operator and show the orthogonality behavior of the eigenfunctions. For the bent waveguides, the asymptotic behavior of the eigenfunctions (i.e., bent modes) dictates the distribution of electromagnetic energy in the radial directions. In this work, we showed mathematically that the asymptotic behavior of the eigenfunctions is proportional to $\frac{1}{\sqrt{r}}$. This information helps to define the appropriate function space and the subsequent mathematical analysis of the wave propagation.

Still, several mathematical questions about the bent waveguide model demand further investigation. e.g., the stability of the model for perturbations in the system parameters, the nature of its pseudospectra, etc. The compactness of the operator for the bent waveguide eigenvalue problem needs to be explored to get more insights into the model. Moreover, one can extend this work to the future 2−D set-up of the bent waveguides.