Graph Theoretic Aspects of Quantum Information Processing

A Thesis submitted by
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“My heart sings at the wonder of my place
In this world of light and life;
At the feel in my pulse of the rhythm of creation
Cadenced by the swing of the endless time.
I feel the tenderness of the grass in my forest walk,
The wayside flowers startle me:
That the gifts of the infinite are strewn in the dust
Wakens my song in wonder.
I have seen, have heard, have lived;
In the depth of the known have felt
The truth that exceeds all knowledge
Which fills my heart with wonder and I sing.”

- Rabindranath Tagore.
(Shantiniketan Patrika, 1924)

আকাশভরা সূর্য-তারা, বিশ্বভরা প্রাণ,
তাহারি মাঝখানে আমি পেয়েছি মোর স্থান,
বিস্ময়ে তাই জাগে আমার গান ॥
অসীম কালের যে হিল্লোলে জোয়ার-ভাঁটায় ভুবন দোলে
নাড়ীতে মোর রক্তধারায় লেগেছে তার টান,
বিস্ময়ে তাই জাগে আমার গান ॥
ঘাসে ঘাসে পা ফেলেছি বনের পথে যেতে,
ফুলের গন্ধে চমক লেগে উঠেছে মন মেতে,
ছড়িয়ে আছে আনন্দেরই দান,
বিস্ময়ে তাই জাগে আমার গান ।
কান পেতেছি, চোখ মেলেছি, ধরার বুকে প্রাণ ঢেলেছি,
জানার মাঝে অজানারে করেছি সন্ধান,
বিস্ময়ে তাই জাগে আমার গান ॥

Dedicated to those,
who will canvas quantum information theory
and its combinatorial perspectives,
in future.

7
Conclusion

7.1 Summary

The interface of quantum mechanics and graph theory is an emerging field of research. It utilizes the tools of mathematics, physics and computer science. We began this thesis with a very brief survey at the interface of the two fields: graph theory, and quantum mechanics. Interesting topics in this field include quantum graphs, tensor network, and graph C* algebras, among others. In this thesis, we have exploited the concept of graph Laplacian quantum states.

Given a Laplacian matrix of a graph there is a corresponding density matrix. We considered this quantum state as a graph Lapacian quantum state. In general it is a mixed state. This work is bidirectional. On one hand, we have investigated properties of a class of mixed quantum states in terms of graphs, such as, unitary operations, quantum entanglement and quantum discord. On the other hand, we have employed tools and techniques of quantum information theory in graph theoretical problems.

In the chapter 3, we have discussed a number of local unitary operations on graphs. Quantum gates are unitary operations. They act as local unitary operators when we apply them in a multi qubit system. An n qubit quantum state is represented by a graph consisting of 2^n vertices. Action of the quantum gate is equivalent to a set of changes in the edge set of the graph. Here, we have discussed procedures of X, Y, Z, H and $CNOT$ gate operations in terms of graphs. Also, we have developed graph theoretic procedures for Bell state generations.

We have discusses quantum entanglement, in chapter 4. Partial transpose is a tool of linear algebra used in entanglement detection. Using the idea of partial transpose we have introduced the idea of partially symmetric graphs. In higher dimensional Hilbert spaces, $\mathcal{H}^{(m)} \otimes \mathcal{H}^{(n)}$, partial symmetric graphs generated separable graph Laplacian quantum states. Also, we have explored the idea that graph isomorphism may generate mixed entangled states from mixed separable states.

Chapter 5 is dedicated to quantum discord. A quantum state has zero discord if the blocks of its density matrix form a family of commuting normal matrices. We have employed this idea in the graph theoretic context. We have constructed the condition on graphs for commutativity and normality. These conditions are in terms of neighbourhood of vertices. We classified quantum states with zero and non-zero discord using the above graph theoretic conditions. Then we generated a measure of quantum discord using graph theoretic parameters.

The idea of graph theoretic partial transpose and graph theoretic counterpart of commuting normal matrices was used in chapter 6 to generate a family of non-isomorphic cospectral graphs. We have shown that graph theoretical partial transpose is a useful tool for generating such classes of graphs. We have also attempted to solve a system of Lyapunov equations in this chapter.