

# Acknowledgements

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## List of Symbols

<i>Symbol</i>	<i>Description</i>
$ 0\rangle,  1\rangle$	Qubits
$ \psi\rangle$	An arbitrary Quantum State
$\alpha$	State parameter
$\beta$	State parameter
$\oplus$	Modulo addition
$\sigma_x, \sigma_y, \sigma_z$	Pauli operators
$\rho$	Density operator
$T_r$	Trace
$I$	Identity operator
$\vec{r}, \vec{s}$	Polarization vector
$T_r^A$	Partial trace with respect to qubit A
$c_{ij}$	Complex coefficients for arbitrary quantum states
$H$	Hilbert Space
$E$	Entanglement of formation
$U$	Unitary operator
$ \phi^\pm\rangle,  \psi^\pm\rangle$	Bell states
$\lambda_i$	Eigen values/Schmidt coefficient
$C$	Concurrence
$S$	von-Neumann entropy
$\rho^*$	Complex conjugate of density matrix $\rho$
$E_N$ or $N$	Negativity
$\tau$	3- tangle
$\sigma$	Sigma- A measure of degree of entanglement in three qubit states
$B$	Bell operator
$M$	Mermin operator
$S_v$	Svetlichny operator
$I(\rho)$	Mutual information
$D_A(\rho)$	Quantum discord
$D_G(\rho)$	Geometric discord
$\lambda_{max}$	Maximum eigenvalue
$E_l^i$	Kraus operator for noisy channels
$\gamma$	Magnitude of decoherence
$ \Psi\rangle$	An arbitrary pure quantum state
$\Lambda_i^{wk}$	Weak measurement operator
$\Lambda_i^{wkr}$	Weak measurement reversal operator
$\eta_i$	Strength of weak measurement

$\eta_{ri}$	Strength of weak measurement reversal
$M(\rho)$	Horodecki's measure
$\mu$	Strength of weak measurement for depolarising channel
$\mu_r$	Strength of weak measurement reversal for depolarising channel
$F_{\max}$	Maximum fidelity for teleportation
$f_{\text{ent}}$	Singlet fraction
$C_{\max}$	Maximum channel capacity for dense coding
$B_C$	Bell-Cumulant operator
$C_\rho$	Correlation matrix
$C_{nm}$	Correlation Coefficients
$\Omega_0$	Set of zero discord states
$S_C$	Modified Svetlichny operator
$\delta$	Relative phase
$\xi$	Relative phase
$ \chi\rangle$	Chi State

## List of Abbreviations

<i>Abbreviation</i>	<i>Full form</i>
BB84	A key distribution protocol developed by Charles H. Bennett and Gilles Brassard in 1984
BSM	Bell-state Measurement
CHSH	John Clauser, Michael Horne, Abner Shimony, and Richard Holt
EPR	Einstein Podolsky and Rosen
FEF	Fully Entangled Fraction
GHZ	Greenberger-Horne-Zeilinger
LOCC	Local Operation and Classical Communication
NMR	Nuclear Magnetic Resonance
Ph.D.	Doctor of Philosophy
QC	Quantum Computation
QIC	Quantum Information and Computing
QIP	Quantum Information and Processing
QKD	Quantum Key Distribution
RSA	Ron Rivest, Adi Shamir and Leonard Adleman
SLOCC	Stochastic Local Operation and Classical Communication

