

Introduction

1.1 INTRODUCTION

The second decade of 21st century has been witnessing more and more connected systems. Emerging applications like Smart Grids, Internet of Things (IOT) will create a large eco-systems of connected entities. For many such systems and sub-systems we need a distributed control. The study of distributed control systems is motivated from its applications in diverse domains of engineering and sciences which includes control of complex networks, vehicle coordination control, distributed networked control, mobile sensor networks, synchronization of oscillators, biological networks, social networks, power networks and robot formation in robotic networks. In all these applications network plays a prominent role in deciding system performance. These distinct applications of distributed systems require different modeling techniques to capture network dynamics and therefore it needs different design tools and analysis techniques. A complex network can be defined as a network consisting of several nodes which interact with each other to share information or exchange physical quantities. Network comprises of nodes and edges. A node represents an entity whereas the edges embed connections or interactions between the nodes. Nodes can be linked to various systems from diverse fields such as genes in a biological network, sensors in a detection system, vehicles in a traffic system and persons or individuals in a social network. The attribute of a node has a state which describes position coordinates of a sensor or a robot, an opinion of a person, expression of protein by a gene etc. In distributed systems, different nodes (agents) connected through links and they work collectively to achieve overall objectives of a given system. There are many applications which can be modeled as collection of homogeneous nodes and interconnections of different nodes occur though links of different weights. These systems fall in a broad category of complex networks [Rahmani et al., 2009; Liu et al., 2011; Yan et al., 2012; Nepusz and Vicsek, 2012; Pasqualetti et al., 2014; Ruths and Ruths, 2014; Hou et al., 2015; Frasca et al., 2015].

Network has been playing an increasing role in several systems around us. A few examples of networked systems are cellular network, spreading of diseases (like ebola) at a massive scale [Pastor-Satorras and Vespignani, 2001], gene regulatory networks [Lezon et al., 2006] etc. Some of the artificial networks which have become common are the Internet [Faloutsos et al., 1999], World Wide Web [Adamic, 1999]. Similarly, there are many other examples of artificial networks like power grids [Wang and Chen, 2003], transportation systems [Guimera et al., 2005], economic networks [Kirman, 1997], communication networks [Bartoszewicz, 2006], social networks [Wasserman and Faust, 1994] etc. These examples arise in many disciplines such as Engineering, Biology, Chemistry, Physics, Sociology and Geography etc. The entities of these real world systems are connected in a complex manner with each other such that they are broadly classified as the complex networks. Basically, a complex network represents logical and/or physical interconnected components such that they interact with each other. Network topology plays a significant role in overall operation and functioning of the system. For instance, for the power networks, performance, robustness to different kinds of perturbations (like attacks and failures), all strongly depend on the network topology [Wang and Chen, 2003]. Some practical applications of the physical and social networks are documented in [Buchanan, 2003].

Graph theory is one of the important tools among many to model complex networks. As

per the documented history, the first real world problem solved using the graphs is the Konigsberg bridge problem in 1735 by Leonard Euler [Euler, 1741]. The mathematical reasoning behind the problem led the foundations of graph theory. A psychologist by name Jacob Moreno in 1933 has given an example which is related to the field of sociology and called Sociogram. He explained about a network which represents a group of people and how the interpersonal structure of a group of people looks like. The fundamental concepts of graph theory are used in many disciplines. After the introduction of random graphs by the two Hungarian mathematicians Paul Erdos and Alfred Renyi [Erdds and R&WI, 1959; Erdos and Rényi, 1960], led many scientific discoveries and key results [Barabási, 2003].

The works of Watts and Strogatz in 1998 [Watts and Strogatz, 1998], and Barabasi and Albert in 1999 [Barabási and Albert, 1999] proposed the ideas of scale-free and small-world properties. In particular, three conclusions have given impetus to research in the field of complex networks: Watts and Strogatz's investigation of larger clustering and small diameter leads to the discovery of small-world phenomenon in real-world networks [Watts and Strogatz, 1998], Barabasi and Albert's investigation of hub nodes which leads to scale-free phenomenon in real-world networks [Barabási and Albert, 1999] while Newman's description of community structures demonstrated its existence in real-world networks [Newman, 2001]. Complex networks research is generally classified into two major fields i.e. network modeling and network analysis. The complex network has traditionally been modeled as the Erdos-Renyi random graph [Erdds and R&WI, 1959; Erdos and Rényi, 1960; Erdős and Rényi, 1961]. The analysis of complex network helps in characterizing the behaviors of large scale network structure.

In several real-world applications including large systems, sets of nodes require to agree upon some quantities of interest. As a result, in [Olfati-Saber and Murray, 2004; Olfati-Saber et al., 2007; Li et al., 2010; Pu et al., 2012; Pasqualetti et al., 2012; Su et al., 2013; Wang et al., 2014; Yang et al., 2014; Wang et al., 2015; Yang et al., 2016], the authors discussed the consensus problem (or agreement problem) for the leader-follower type networks. In addition, the analysis of the robustness of a distributed discrete-time complex network based on leader-following consensus approach is presented in [Yang et al., 2014]. This concept has been used in systems with many nodes which are connected in distributed manner in different systems to arrive at consensus. These nodes reach to a consensus value (or agreed value) depending on target of a system and connectivity and the initial state of the nodes. The state of nodes is represented by a vector x , where any entry $x_i \in \mathbb{R}$ represents the state value of node i . At the beginning, the nodes may have some state value depending on its previous state and the network connectivity after some time, they come to an equilibrium (or consensus). Mathematically the consensus is defined as $\|x_i(t) - x_j(t)\| \rightarrow 0 \forall i, j$ at time, when $t \rightarrow \infty$.

One of the fundamental needs in complex networks is the ability to control the network by using a few control nodes (driver nodes). There are several applications where the nodes are homogeneous i.e. they are identical. The dynamics on network may be non-linear time varying depending on a particular application. However, in many applications it results in linear dynamics or it is possible to linearize the actual non-linear dynamical equations without loss of generality. For linear dynamics, the ability to control the network is measured by controllability property. There are two major coupled issues here the first is selection of control nodes (driver nodes) and the second is ensuring controllability of the network. In recent years network controllability is well explored in the literature for example see works of [Liu et al., 2011; Yan et al., 2012; Lombardi and Hörnquist, 2007; Menichetti et al., 2014; Frasca et al., 2015; Rahmani et al., 2009]. Controllability of the complex networks can be investigated by modeling the networked system as a Linear Time Invariant (LTI) system. In [Ding et al., 2016], the authors have presented the relation between the network structure and its controllability index for the directed networks with an optimized control design.

A first significant contribution on network controllability and selection of driver nodes was made by Liu et al. in 2011. This work uses graph theoretic concepts to characterize controllability and selection of driver nodes. The classic notion of controllability is based on matrix rank is difficult to apply for complex networks as, in general, dimension of these systems is very large. This has created a renewed interest in controllability of complex networks using tools from the graph theory. The work proposed by Liu et al. in 2011 uses structural controllability property and provides a method to select driver nodes which ensures controllability. However, the work proposed by Liu et al. in 2011 gives only binary answer about network controllability, it does not explain about difficulty in control and in particular difficulty associated with a given set of driver nodes.

Due to large size of a network it is difficult to determine number of nodes needed to control the network. The seminal work to identify driver nodes was proposed in [Rahmani et al., 2009; Liu et al., 2011]. The minimum number of driver nodes required to control a complex network is fixed and the relevant theory is documented in [Liu et al., 2011; Yuan et al., 2013; Yazicioglu et al., 2016; Haghighi and Cheah, 2016]. In [Yuan et al., 2013], authors developed a theory of the maximum multiplicity of the eigenvalues of the Adjacency matrix to identify the minimum number of driver nodes required to control the entire network with arbitrary structures. In [Pasqualetti et al., 2014], authors proposed a trade-off between the driver nodes and the control energy as a function of the network dynamics using the smallest eigenvalue of the controllability Gramian. In [Ding et al., 2016; Gao et al., 2014], authors discussed the network controllability with linear dynamics of real-world networks such as the Erdos-Renyi and scale-free networks. To control a network, in general, there are several combinations exist to choose driver node(s). Thus set of driver node(s) is not unique and poses an important question of selection of driver nodes when multiple options exist. To address this challenge, recently, some techniques involving the selection of set of right driver nodes, based on an average control energy bound [Summers et al., 2016], graph distances between the nodes [Yazicioglu et al., 2016], optimal leader selection based on Joint Centrality [Fitch and Leonard, 2016], and optimal actuator placement based on Control Energy Centrality [Tzoumas et al., 2016] have been proposed.

It has been an established fact that behavior of the system has a strong relationship with topological structure of the networks. Therefore, it is imperative to relate structural properties of network with the system behavior. The correlation between control indices and properties of the network was presented in [Ruths and Ruths, 2014]. In several applications it is necessary to understand the influence of a node (or a set of nodes) on a specific node (or a set of specific nodes). When the system dynamics are Linear Time-Invariant (LTI), transfer function between input and output nodes models input-output behavior of the system. An excellent study to understand input-output behavior, for consensus problem, using transfer functions is presented in [Briegel et al., 2011]. In addition, Yoon et al. in [Yoon and Tsumura, 2011] derived a formula of the transfer function for a specific type of consensus network e.g. cyclic network and considering different reference nodes. Moreover, in [Briegel et al., 2011] authors presented the network properties such as shortest distance between two nodes interlaced with the Gain Factor and relative degree of the transfer function with Laplacian matrix of the unweighted networks. There are some other interesting works which establish relationship between network structure and dynamic properties of the system [Torres and Roy, 2015; Yoon, 2016].

While studying effect of one node on another node, sensitivity transfer function captures robustness of input-output pairs. In [Tonetti and Murray, 2010], authors derived the sensitivity transfer functions between a pair of nodes of randomly connected networked systems, using the signal-flow graph technique which is known as Mason's Direct Rule [Mason, 1953] and S. Tonetti et al. in [Tonetti and Murray, 2010], investigated the sensitivity transfer functions of the closed-loop controlled multi-agent systems with limits of the performance index. Recently, authors of [Boas et al., 2010; Passos et al., 2014] described the sensitivity function for the networked systems.

Many researchers are developing new theories that are common to different kind of networks such as power, biological, technological and social networks. Several models have been studied and recommended to depict the behavior of these real world networks as means to understand the structure of complex systems. Power system is getting renewed interest partly because of need of smart grid and partly due to the requirement of energy efficient transmission and distribution system. However, this causes the power system to operate under high stress which leads to instabilities in the systems [Li et al., 2015b]. To deal with this problem, networked control theory can be applied where minimum number of driver nodes can be selected in such systems, which acts as self healing distributed networked system which improves system reliability and efficiency. However, these set of minimum number of driver nodes, required to control the networked system, are calculated by using the maximum matching algorithms [Liu et al., 2011] and the minimum control energy [Pasqualetti et al., 2014].

In what follows next, different techniques to select driver nodes are briefly reviewed.

1.2 CHARACTERIZATION AND SELECTION OF DRIVER NODES

In this section a review of literature on identification of minimum number of driver nodes needed to control the network and selection of right set of driver nodes, based on certain criteria, in the complex networks is presented. Techniques reported for identifying the minimum number of driver nodes required to control the network are classified. Selection of set of driver nodes among them and the concept of each technique is described briefly along with its merits and limitations.

There are several methods exist to identify the set of minimum number of driver nodes (denoted by N_D) to control a complex network and satisfy the Kalman rank condition. These methods are briefly reviewed in the following sections.

1.2.1 Techniques to Find Minimum Number of Driver Nodes

This section presents the brief overview of methods or algorithms reported in the literature to control a network using minimum number of driver nodes.

(a) *Minimum Input Theory for Digraphs Based on Structural Controllability*

The concept of structural controllability of the linear system was introduced by Lin in 1974 [Lin, 1974]. The author considered that the elements of matrices A and B of the LTI system are either zero or some non-zero free parameters. The parameters of system matrix A are generally not known. Only the zero values of the matrix are known. These zero values represent that there is no connection between the corresponding nodes. A structurally controllable system is one in which the free parameters in A and B are fixed as well as satisfying the Kalman rank condition [Kalman et al., 1969]. The structural controllability is a generic property of the system. A structurally controllable system is either controllable or the controllability is induced in it after (slightly) modifying the weights of certain connections such that it remains controllable with respect to the significant changes of the link weights [Liu et al., 2011]. A strongly structurally controllable system remains controllable for any values (except zero) of the free parameters of the system [Mayeda and Yamada, 1979].

Figure 1.1 illustrates the concept of Controllable, Structural Controllable and Strong Structural Controllable networks with the help of the following networks [Liu et al., 2011].

There are several systems where system matrix is modeled as Adjacency matrix [Liu et al., 2011; Pasqualetti et al., 2014]. The Adjacency matrix, input matrix and Controllability Matrix (CM) of the system shown in Figure 1.1(a) can be written as

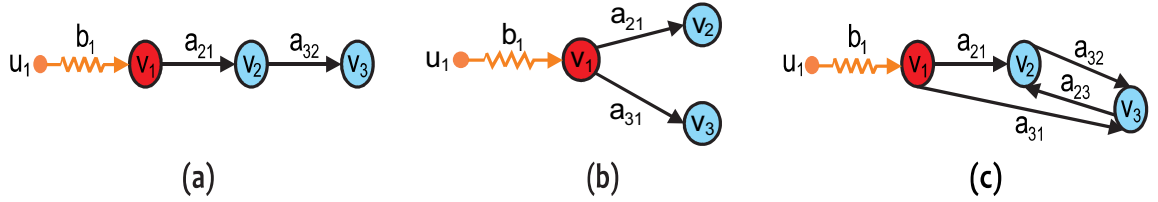


Figure 1.1 : Single input controllable networks (a) Network is Controllable, Structural Controllable and Strong Structural Controllable, (b) Network is not Controllable through v_1 node and, (c) Network is Controllable and Structural Controllable (Source: Liu et al, 2011)

$$A = \begin{bmatrix} 0 & 0 & 0 \\ a_{21} & 0 & 0 \\ 0 & a_{32} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ 0 \\ 0 \end{bmatrix}, \quad CM = b_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{21} & 0 \\ 0 & 0 & a_{32}a_{21} \end{bmatrix} \quad (1.1)$$

It is observed that if $\text{rank}(CM) = 3 = n$ and the values of the edge weights (i.e. a_{21} , a_{32} and b_1) are non-zero then the network is always controllable. Hence, the network is called structural controllable as well as strong structural controllable because the system controllability is independent of the edge weights.

Also, the Adjacency matrix, input matrix and CM of the system shown in Figure 1.1(b) can be written as

$$A = \begin{bmatrix} 0 & 0 & 0 \\ a_{21} & 0 & 0 \\ a_{31} & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ 0 \\ 0 \end{bmatrix}, \quad CM = b_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{21} & 0 \\ 0 & a_{31} & 0 \end{bmatrix} \quad (1.2)$$

It is observed that the $\text{rank}(CM) = 2 < n$, hence system is uncontrollable. It is worth to note that the value of $\text{rank}(CM)$ is independent of the values of the edge weights (i.e. a_{21} , a_{31} and b_1). Hence the network is not controllable.

Similarly, the Adjacency matrix, control input matrix and CM of the system shown in Figure 1.1(c) can be written as

$$A = \begin{bmatrix} 0 & 0 & 0 \\ a_{21} & 0 & a_{23} \\ a_{31} & a_{32} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ 0 \\ 0 \end{bmatrix}, \quad CM = b_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{21} & a_{23}a_{31} \\ 0 & a_{31} & a_{32}a_{21} \end{bmatrix} \quad (1.3)$$

It can be seen from the above equation that the $\text{rank}(CM)$ depends on the expression $\Omega = (a_{32}a_{21}^2 - a_{23}a_{31}^2)$. For non-zero values of Ω , $\text{rank}(CM) = 3 = n$, the network is controllable whereas for $\Omega = 0$ and $\text{rank}(CM) = 2 < n$, network becomes uncontrollable. Any slight change in free or independent parameters will make $\Omega \neq 0$. This makes the network controllable. Hence, the network is structurally controllable but not strongly structurally controllable.

The works on structural controllability of complex networks have been reported in the literature [Gao et al., 2014; Menichetti et al., 2014; Wang et al., 2016]. In [Gao et al., 2014], authors presented structurally controllability approach based on ‘ k -walk’ theory for target control to find out the minimum number of driver nodes in the system. The same approach based on in-degree and out-degree has been reported in [Menichetti et al., 2014]. While in [Wang et al., 2016], the reachability concept has been presented.

Minimum number of the driver nodes required to fully control an arbitrary network can be calculated using the theory of full row rank of Controllability Matrix (CM). This theory can be implemented to control a network if the exact weight of each link presents in the network is known. However, these weights for most real networks are unknown. The brute-force search algorithm requires to compute the rank of CM for $2^n - 1$ distinct combinations with all known weights. The calculation to find the $rank(CM)$ is computationally complex task for a large network. Liu et al. in 2011 developed a minimum input theory to efficiently characterize the structural controllability of directed networks using a minimum set of driver nodes to control the network. This theory does not require the knowledge of link weights.

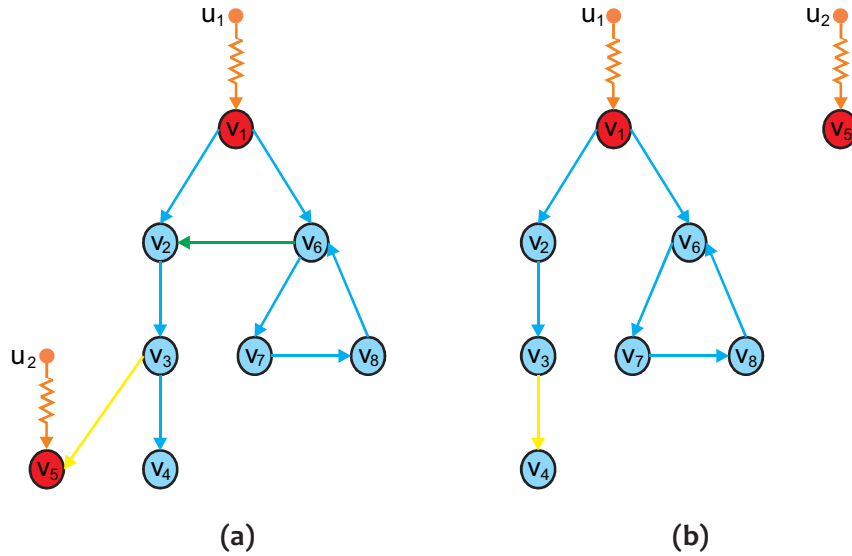


Figure 1.2 : A schematic diagram of controlling of a directed network, (a) Digraph $G(A, B)$ and, (b) Cactus of the network (Source: Liu et al, 2011)

Figure 1.2 illustrates the concept of minimum input theorem using a network. For digraph the matching is a set of directed edges without common heads or tails and in undirected case it is a set of edges without common nodes. A node is said to be matched either it is an endpoint of the edge among the edges in the matching set (for digraph) or edges of this node belongs to the matching set (for undirected), otherwise it is unmatched. For a network shown in Figure 1.2(a), the unmatched nodes (shown in red color) are $\{v_1, v_5\}$ and matched nodes (shown in blue color) are $\{v_2, v_3, v_4, v_6, v_7, v_8\}$. From the above theory, it can be inferred that the minimum number of driver nodes is the unmatched nodes of a network. Hence, the minimum number of driver nodes is two i.e. node v_1 and node v_5 . However, this combination of minimum number of driver nodes is not unique. For instance, in this network a set of driver nodes $\{v_1, v_4\}$ also qualify for the combination of the minimum number of driver nodes. The control inputs (shown in orange color) directly apply to this set of driver nodes to fully control the network. The critical links (shown in blue color) without which the entire network cannot be controlled for a given maximum matching. Redundant links (shown in green color) are the links which can be removed without affecting the controllability of the network. The standard link (shown in yellow color) and the other link like $(v_3 \rightarrow v_4)$ are

the links whose removal can change the controllability of the network but retain the same set of minimum number of driver nodes. The edge $v_1 \rightarrow v_6$ is also called prominent edge of the network with the corresponding bud (an elementary cycle with an extra edge which end, but does not start, in a node of the cycle) $v_1 \rightarrow v_6 \rightarrow v_7 \rightarrow v_8$ [Liu et al., 2011]. A cactus (minimal structure which have neither dilations (large subset of nodes) nor inaccessible nodes (no direct path reaching from input node to it)) node set contains all node of the corresponding network [Liu et al., 2011]. A stem (an elementary path starting from an input node) covering all nodes of the network is a cactus as well as a set of disjoint stems with buds covering all nodes of the network is also cactus [Liu et al., 2011]. The Figure 1.2(b), shows a cactus for the network of the Figure 1.2(a), which covers the disjoint set of stems i.e $u_2 \rightarrow v_5$ and $u_1 \rightarrow v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_4$, and bud $v_1 \rightarrow v_6 \rightarrow v_7 \rightarrow v_8$.

The Hopcroft-Karp algorithm [Hopcroft and Karp, 1973] (executes the maximum matching algorithm, which is the foundation of minimum input theorem) is used to find the maximum matched nodes for the sparse networks. It has the worst performance of $O(En)$, where E represents the number of edges and n is the number of nodes in the network. For the exact given link weights, the requirement of free or independent parameters of structural matrices may not be met. Hence, it lacks a reliable tool to measure the minimum number of driver nodes for undirected weighted and unweighted networks [Yuan et al., 2013].

(b) Exact Controllability Theory

Yuan et al. in 2013 presented an exact controllability framework to find the minimum number of driver nodes of the complex networks. This framework is based on the arbitrary structures and link weights, including unweighted, weighted, undirected, and directed networks without or with self-loops. The presented framework is more general compared to structural controllability framework.

Yuan et al. in 2013, proved based on Popov-Belevitch-Hautus (PBH) rank condition, that the minimum number of independent driver nodes is equal to the maximum geometric multiplicity of eigenvalues of the Adjacency matrix for a directed network. For undirected networks the minimum number of independent driver nodes is equal to the maximum algebraic multiplicity of eigenvalues of matrix A . For regular networks (each node has exactly the equal number of edges), the exact controllability can be calculated using analytical relations. For real-world weighted complex networks with distinct node-degree distributions, the exact controllability can efficiently be estimated by numerical computations. The minimum set of driver nodes is investigated by elementary transformations using the exact controllability concept.

The PBH rank condition states that a network is fully controllable if and only if $\text{rank}(\zeta I_{n \times n} - A; B) = n$ is satisfied for a complex number ζ , where $I_{n \times n}$ represents identity matrix of the dimension $n \times n$.

The minimum number of driver nodes N_D for an arbitrary network structures and link weights is determined by the maximum geometric multiplicity $\mu(\lambda_i)$ of the eigenvalue λ_i of Adjacency matrix A : $N_D = \max_i(\mu(\lambda_i))$ and $\lambda_i, i = 1, 2, \dots, l \leq n$, represents the distinct eigenvalues of A . For an undirected networks with arbitrary link weights, N_D is determined based on the maximum algebraic multiplicity $\delta(\lambda_i)$ of λ_i : $N_D = \max_i(\delta(\lambda_i))$ where $\delta(\lambda_i)$ represents the eigenvalue degeneracy of matrix A .

The calculation of algebraic and geometric multiplicities for large networks is computationally intensive task. In [Yuan et al., 2013] authors presented a formula to determine the minimum number of driver nodes for the sparse as well as dense networks. The complex networks representing the real-world applications are typically sparse networks. For a large (having large number of nodes n and links) sparse network, with a small fraction of self-loops, N_D is given by the rank of A :

$N_D = \max\{1; n - \text{rank}(A)\}$. For a large densely (zero values in A matrix increases with n) connected network having identical link weights (W) and self-loops: $N_D = \max\{1; n - \text{rank}(WI_{n \times n} + A)\}$. Yuan et al. in 2013, also presented a general method for identification of a minimum set of driver nodes which needed to control the entire network.

(c) Source and Sink Nodes Based

Ruths et al. in 2014 extended the concept of degree distribution in a network, and proposed a mathematical formula to determine the minimum number of driver nodes needed to control the entire network: $N_D = N_S + N_E + N_I$, where N_S represents number of source nodes, N_E is number of external dilation points i.e. $N_E = \max\{0; N_T - N_S\}$, where, N_T corresponds to the number of sink nodes and N_I is the internal dilation points in the network. Additional stems, each one requiring a separate control to control the entire network are introduced at these connecting points. The surplus of sink nodes leads to the external dilation and the remaining nodes form internal dilation points. In a special case with $N_S = N_E = N_I = 0$ such as a cycle network, the cardinality of the set of minimum number of driver nodes is one. Ruths et al. in 2014 also established that the lower bound on the minimum number of driver nodes can be approximated based on the counting of source and sink nodes in a network. In this approach, computational steps required are of the order of edges. Hence, an improvement over the maximum matching algorithm has been achieved, which gives the exact minimum number of driver nodes in the network.

(d) Control Energy Based

In [Pasqualetti et al., 2014], the authors described three problems i.e., (i) the relationship between the number of control nodes and control energy where energy is bounded and it depends on the system dynamics (i.e., weights of edges and network topology), (ii) a controllability metric is the function of the control energy, and (iii) performance based on an open-loop control strategy. Bullo et al. in 2013 described the control limitations and few properties for symmetric and Schur stable networks which are: (i) some networks can be controlled with continual energy which is independent of the network dimension if the number of control nodes is fixed among the total number of the network nodes, and (ii) as the controllability properties of cluster network and their coupling strength affects the control energy results the clustered networks may be easier to control. In [Sun and Motter, 2013], authors investigated the physical implementation of control signals as well as the properties of the resulting minimal energy control trajectories.

(e) Other Methods

The selection of the driver nodes in brain networks has been presented by the authors in [Bullmore and Sporns, 2009]; the recent challenges and direction of future in the control of biological networks have also been presented. The number of driver nodes related to controllability properties for distinct fields such as physical, social, biological and technological systems presented in [Nepusz and Vicsek, 2012]. In [Ruths and Ruths, 2016], authors presented a new index known as Minimum Control Counts (MCC) to count exact number of driver nodes in random networks.

1.2.2 Techniques for Selecting Set of Driver Nodes

The minimum number of driver nodes required to control a networked system is fixed [Liu et al., 2011]. This set of driver nodes can be obtained with the help of existing theories [Liu et al., 2011; Yuan et al., 2013; Yazicioglu et al., 2016; Haghighi and Cheah, 2016]. However, the driver nodes are not unique and generally many possibilities exist to choose the driver nodes. Finding an optimal set of driver nodes is one of the major challenges to control complex networks. Controlling driver nodes eventually leads to control of the entire networked systems in a desired manner. The aforementioned concept has been illustrated through a directed network of seven nodes showing different choices of driver nodes with different choices of output nodes as shown in Figure 1.3.

The different choices of the driver node and control node are based on the controllability and observability of the system.

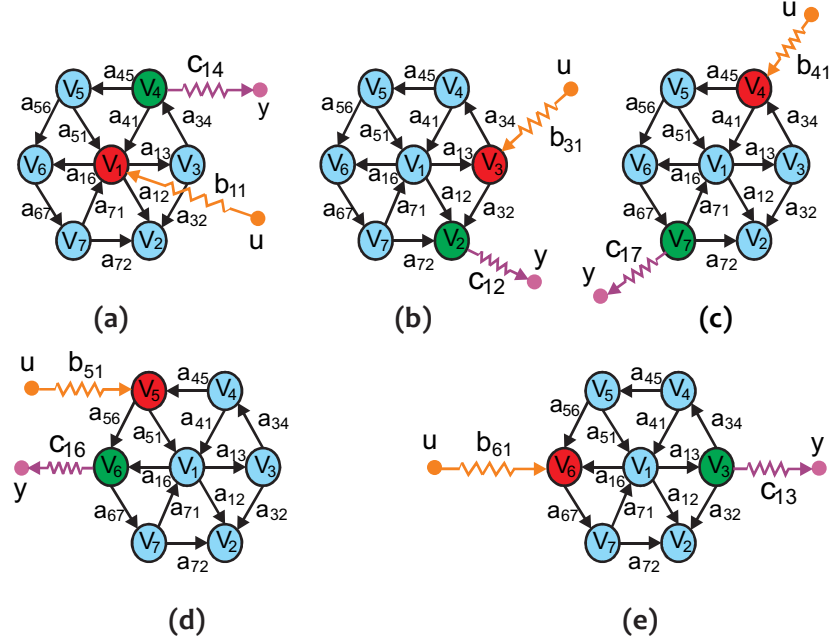


Figure 1.3 : Seven nodes network showing different choices of driver nodes and output nodes (a) $\{v_1, v_4\}$, (b) $\{v_3, v_2\}$, (c) $\{v_4, v_7\}$, (d) $\{v_5, v_6\}$, and (e) $\{v_6, v_3\}$

The existing ranking scheme of driver nodes reported in the literature which quantifies the importance of a driver node in maintaining the controllability of complex networks is discussed. Recently, reported techniques involve the identification of a set of most suitable driver nodes based on graph distances between nodes [Yazicioglu et al., 2016], minimal actuator placement based on an average control energy bound [Summers et al., 2016], and optimal actuator placement based on Control Energy Centrality (CEE) [Tzoumas et al., 2016] in the complex networks.

Therefore, there are several techniques to select a set of driver nodes to control the entire complex networks in the following section, these techniques are reviewed.

(a) Centrality and Network Structure

The control centrality of i^{th} node is equal to the dimension of the controllable subspace when a control input is applied to the i^{th} node of the network only. Authors in [Liu et al., 2012] introduced a theory of control centrality which quantifies the ability of a single node to control a directed weighted network. It is proposed that the control centrality of a node is determined by its hierarchical level or layer index in the network (having no self-loop). In [Clark et al., 2012], authors presented a new metric, known as the Graph Controllability Index (GCI). This is defined as the fraction of network nodes that are controllable using the leader set and select an optimal set of driver nodes considering the controllability and guaranteed performance. In [Fitch and Leonard, 2016], authors described the optimal leader node corresponding to the minimum steady state error with the external signal. The authors also introduced an information centrality or joint centrality which measures the optimal leader for both the noise-corrupted and noise-free leader cases in the underlying networks. This algorithm has better results compared to the previously published greedy algorithms for the optimal driver node selection. In [Orouskhani et al., 2016], authors described the population-based optimization technique (or cat swarm optimization) to improve the pinning controllability with four optimization tasks: (i) optimizing the feedback gains, (ii) optimizing the

locations of driver nodes, (iii) optimizing the connection weights, and (iv) optimizing simultaneously the feedback gains and locations of the driver nodes. This optimizing technique is applied for the scale-free, real-world, and the small-world networks. The effective ranking method based on the importance of weights of edge between a pair of nodes presented in [Liu et al., 2016]. In [Zhang et al., 2016b], authors presented a method related to the controllability of two-layer networks and found the optimal driver node using high weight for central nodes of the one layer and controlled the remaining layers in the networks.

(b) Control Range Based

Wang et al. in 2012, proposed a concept of control range to quantify the importance of a node which help in guaranteeing the structural controllability of a network. Proposed concept is similar to the control centrality method reported in [Liu et al., 2012]. The control range of a node i equals to the number of the reachable nodes from this node in cacti of the network. In this study it has been observed that the nodes with a low degree may have a high control range and the control range of nodes is correlated to the network's degree distribution. The detailed analysis of this method has been reported in [Wang et al., 2012].

(c) Control Capacity and Backbone Based

Jia and Barabasi in 2013, proposed a method to measure the control capacity of a single driver node which helps in controlling the network by counting its frequency of appearance in the minimum set of driver nodes. It is also observed that the control capacity of a given driver node is independent of out-degree and decreases with its in-degree. The detailed analysis of this method is documented in [Jia and Barabási, 2013].

In [Ding and Lu, 2015], authors developed a method of control backbone similar to that reported in [Jia and Barabási, 2013] which quantifies a node's importance that helps in maintaining the structural controllability of the network. The control backbone of the node i is obtained by computing the number of times it appears in the minimum set of driver nodes. The normalized control backbone of a node i is denoted by cb_i . If the control backbone of a node i equals to one i.e. $cb_i = 1$ then the node i will appear in all sets of minimum driver nodes. Without applying an input to these driver nodes, the structural controllability of the network can not be guaranteed. This set is called critical set in the network. If the control backbone of a node i is zero i.e. $cb_i = 0$ then the node i does not appear in any sets of minimum driver nodes. The structural controllability of network can be maintained without applying an input to this node. Hence, this set known as redundant set in the network. For the higher value of control backbone of a node, the structural controllability of the network remains intact. The detailed analysis of this method has been reported in [Ding and Lu, 2015].

(d) Control Energy Based

In [Bof et al., 2016], authors described a relationship between controllability and network centrality based on bounded energy and selected an optimal set of driver nodes based on this centrality. Authors in [Bianchin et al., 2015], presented a study related to the controllability indice using the worst-case control energy which is the function of diameter of the networks. In [Li et al., 2015a], authors proposed a projected gradient method used to find the optimal location of actuators and sensors with minimum cost of control energy in the networks. In [Summers et al., 2016], authors presented a study related to the basic problem of sensor and actuator placements. Choose a subset from a finite set of possible placements to optimize real-valued controllability and observability Gramians of the network. The mapping from placements to scalar functions of the associated Gramian is either a modular or submodular set function. In [Pasqualetti et al., 2014; Tzoumas et al., 2016], authors addressed the problem of minimal actuator placement in a linear

system limited to the average control energy bound. Based on the work of Olshevsky in 2015 [Olshevsky, 2015], authors proved that this is NP-hard. They provided an efficient algorithm for a given range of problem parameters, approximates up to a scaling factor of $O(\log n)$, with n being the network size, any optimal actuator set that meets the same energy criteria. This is the best approximation factor which can be achieved in polynomial time. In the worst case this is proved to be supermodular. Authors also focused on the related problem of cardinality-constrained actuator placement for minimum control effort, with the selection of the optimal actuator set minimizing an average input energy metric. The detailed analysis of these methods has been reported in [Bof et al., 2016; Bianchin et al., 2015; Li et al., 2015a; Olshevsky, 2015; Summers et al., 2016; Tzoumas et al., 2016].

(e) Network Properties Based

In [Tanner, 2004], authors derived the necessary and sufficient conditions for selecting an optimal driver node based on the nearest neighbor rules. In [Yazicioglu et al., 2016], authors presented a graph theoretic perspective result which is related to the controllability of leader-follower system for the distance between the pair of nodes in the network. More specifically, authors presented a graph topological lower bound on the rank of $\mathbf{CM}$. This lower bound is tight, and it is applicable to random networks with coupling weights, network topologies, and number of leaders. In [Patterson et al., 2016], the main objective of the authors is related to the selection of optimal k -leader in consensus networks under two performance measures (i) convergence rate to a consensus value, and (ii) formation coherence limited to additive perturbations, as quantified by the steady-state variance of the deviation from the desired trajectory. In both cases, an optimal set of leaders can be established using an exhaustive search for all possible leader sets; however, this approach is not applicable to large networks.

The above listed methods quantify the importance of a single node in maintaining the structural controllability of the network.

1.3 MOTIVATION

The behavior of the networked system has a strong relationship with topological structure of the networks. These relations provide an insight into a hidden connection between system behavior and the network structure. Therefore, it is imperative to investigate this connection. In several applications, it is necessary to understand the influence of a node (or a set of nodes) on a specific node (or a set of specific nodes). The structure of the network plays a significant role to understand the effect of one node on any other node in a complex network. Only a few results for consensus problem (considering Laplacian matrix) are available, however, general complex network control is not analyzed for input-output properties and its relation to network structure is unknown. When the system dynamics are LTI, Transfer Function (TF) between a pair of input and output nodes models input-output behavior of the system. The existing works discussed consensus problem (considering uniform link weights) and do not consider a general setup of complex network to establish a relationship between network structure and important indices. In many real-world applications, the uniform link weights are not realistic and does not model practical networks. Therefore, it is imperative to consider non-uniform link weights (weighted Adjacency matrix) between the nodes for more general setup. The minimum number of driver nodes required to control a network is fixed. However, the set of driver node(s) is not unique and poses an important question of selection of driver nodes when multiple options exist. In a complex network, dynamics is captured by Adjacency matrix, and generally, it has unstable eigenvalues. In any practical applications, the actuators cannot have an infinite capability. Therefore, it is imperative to consider this limitation a priori. The limitations of actuators (driver nodes) pose severe challenges to control when the Adjacency matrix (system matrix) has unstable eigenvalues; even stability cannot be guaranteed

in this situation. For unstable eigenvalues of the Adjacency matrix of the network, the region of guaranteed stability in state space is finite when limitations of actuators are considered. This region is known as Region of Attraction (ROA) and generally, described by ellipsoids in the n -dimensional state space.

The proposed work is aimed to select driver nodes such that Region of Attraction (ROA) becomes maximum. Algorithms for continuous-time and discrete-time dynamics are proposed. The proposed work considers actuator (driver node) limitation a priori which is more realistic and practical. Furthermore, the relative position of the control node and output node and network structure has a strong influence on control indices i.e. Gain Factor, DC-Gain, Sensitivity etc. It has been shown how Gain Factor, DC-Gain, non-minimum phase zeros, Sensitivity are related with network structure. In addition, the Sensitivity depends on the Gain Factor as well as DC-Gain. This work investigates the correlation between the network structure and Sensitivity in the complex networks. The proposed work in this thesis helps to understand the role of the network in input-output dynamics. Finally, the proposed theories and algorithms are verified with the help of real-world network examples and using the numerical simulation.

1.4 THESIS CONTRIBUTIONS

The contribution of this thesis is to establish a relationship between the input-output dynamic properties and graph structure. The work presents, how the Gain Factor, DC-Gain and Sensitivity are related to the network properties such as a distance between the control node and output node, diameter of networks and the sum of the product of weights of all shortest paths between the control node and output node of the particular length in the networks. The work also presents, establishes a relationship between Sensitivity and control indices (Gain Factor and DC-Gain) and selection of the optimal set of driver nodes based on the ROA in the complex networks. These relationships provide an insight into structural properties and its influence on system dynamics.

The contribution of the thesis are summarized below:

- Establishes the relationship among Gain Factor, DC-Gain, System Zeros and network properties i.e. distance between the control and output nodes and diameter of networks.
- Establishes the relationship between Sensitivity and network structure in complex networks.
- Proposes algorithms for the selection of an optimal set of driver nodes based on Region of Attraction (ROA) in complex networks for continuous-time and discrete-time dynamics.

1.5 ORGANIZATION OF THE THESIS

This thesis is organized in 7 chapters and the content of each chapter is described, briefly, as follows.

Chapter 1: This chapter presents an introduction to control of complex network, and its behavior, stability, techniques to the identification of the minimum number of driver nodes and selection of the right set of driver nodes among of them of complex networks, a brief review of the literature, the motivation for the proposed work, and contribution and organization of the thesis.

Chapter 2: This chapter provides a brief introduction of complex network modeling, some basic definitions related to graphs and networked control systems, and discusses the preliminary results related to control of complex networks, controllability and observability, sensitivity transfer function and modeling of power networks.

- Chapter 3:** This chapter obtains the relationship between dynamic properties of the system and network structure of complex networks. It show that the sum of the product of weights of all shortest paths between control and output nodes is equal to the Gain Factor of the pole-zero-gain transfer function of the complex network. It also show that DC-Gain for the peripheral nodes of the complex network depends on the distance between the control and output nodes in the network. The existence of non-minimum phase zeros for collocated control and output nodes is discussed.
- Chapter 4:** This chapter investigates the correlation between network structure properties and Sensitivity of the complex networks. It obtains the relationship between the Sensitivity and the network properties such as distance between the pairs of control and output nodes for a random networked system. In particular, the Sensitivity is related to the sum of product of the weights of all possible shortest path between the control and output nodes of the particular length in the networked systems. In this chapter, the effect of Gain Factor and DC-Gain on the Sensitivity transfer function of the networked systems are presented. Finally, the effectiveness of the developed theory has been validated with the help of the standard IEEE test systems such as IEEE-4 bus and IEEE-14 bus power networked systems.
- Chapter 5:** This chapter characterizes the sets of driver nodes of the networked systems with saturated actuators. A mathematical formulation and algorithms are proposed to identify the optimal set of driver nodes to maximize the ROA for a given control law. The Low and High Gain (LHG) technique has been applied to obtain largest possible region. The presented theory is valid for any stabilizing linear state feedback control law. The proposed algorithms can also be applied to the real-world applications of the networked systems such as power networks, robotic networks, biological networks and social networks etc. The effectiveness of the developed theory has been validated with the help of a robotic network.
- Chapter 6:** This chapter considers a network of discrete-time nodes (agents) which can be controlled by different nodes, acting independently. When a discrete-time network can be controlled through several nodes, it is important to understand differences in choosing a particular node as driver node (node which control the entire network). This work considers limited actuator amplitude of driver node and characterizes driver node based on ROA obtained corresponding to a particular chosen driver node. The presented theory and algorithms enable to find an appropriate node such that ROA maximizes. Theoretical developments are verified through numerical simulation.
- Chapter 7:** This chapter concludes the thesis and summarizes the major contributions. It also presents a few recommendations for future research.

1.6 PUBLICATIONS

The work presented in this thesis have been published to peer reviewed journals and conferences. A list of publications, from this work, is as follows:

- 1 **Ram Niwash Mahia** and Deepak M. Fulwani, "On Some Input-Output Dynamic Properties of Complex Networks", IEEE Transactions on Circuits and Systems II: Express Briefs, Vol. 65, Issue 2, pp. 216-220, February 2018
- 2 Mahaveer Singh, **Ram Niwash Mahia**, and Deepak M. Fulwani, "Towards Characterization of Driver Nodes in Complex Network with Actuator Saturation", Neurocomputing, Vol. 201, pp. 104-111, August 2016
- 3 **Ram Niwash Mahia**, Mahaveer Singh, and Deepak M. Fulwani, "Identification of Optimal Set of Driver Nodes in Complex Networked Systems Using Region of Attraction", International Journal of Control Automation and Systems, Vol. 16, no. 1, pp. 97-107, February 2018, (A Springer Journal)

- 4 **Ram Niwash Mahia**, Mahaveer Singh, and Deepak M. Fulwani, “Characterization of Driver Nodes: Network of Discrete-Time Agents”, 14th European Control Conference (ECC’15), Linz, Austria, pp. 622-627, July 15-17, 2015
- 5 **Ram Niwash Mahia**, Mahaveer Singh, and Deepak M. Fulwani, “Algorithms to Select Right Driver Nodes for Multi-Agent Systems”, 10th Asian Control Conference (ASCC’15), Kota Kinabalu, Malaysia, pp. 1-6, May 31-June 3, 2015
- 6 **Ram Niwash Mahia** and Deepak M. Fulwani, “Selection of Optimal Set of Driver Nodes Based on Networked Sensitivity in Complex Networked Systems”, 3rd Indian Control Conference (ICC’17), Indian Institute of Technology Guwahati, Guwahati, India, pp. 332-337, January 4-6, 2017
- 7 **Ram Niwash Mahia**, Mahaveer Singh, and Deepak M. Fulwani, “Selection of Driver Nodes Based on Region of Attraction for Single-Input Complex Networks”, 2nd Indian Control Conference (ICC’16), Indian Institute of Technology Hyderabad, Hyderabad, India, pp. 188-193, January 4-6, 2016

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