Corona Product in Signed Graph Theory: Modelling, Structural Balance, and Control

A Thesis submitted by
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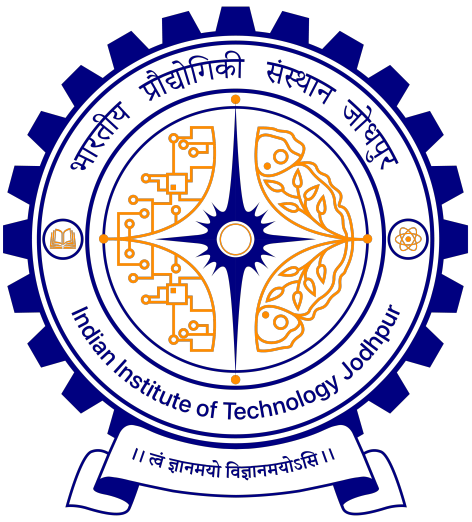
Doctor of Philosophy



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Conclusion and Future work

5.1 CONCLUSION

I have expanded the concept of the generalized corona product, originally defined for unsigned graphs, to include signed graphs. This extension allowed us to explore the structural and spectral properties of signed graphs more deeply. I specifically developed statistical properties such as edge counts and triangle categorizations tailored to the generalized corona product of signed graphs. These properties helped me identify conditions under which the generalized corona product of signed graphs becomes unbalanced. I observed inteeresting property of algebraic conflict for some special signed corona graphs. I also show that by choosing a seed graph I create signed corona graphs with the edge and triangle distribution comparable to real-world physical or abstract networks.

I have also derived explicit expressions for the signed coronal, L-signed coronal, and Q-signed coronal, focusing on specific structured signed graphs. Additionally, I provide formulas for the characteristic polynomial, Laplacian polynomial, and Signless Laplacian polynomial for the generalized corona product of signed graphs, all expressed in terms of their respective coronals.

Furthermore, I have introduced important criteria for identifying cases where the generalized corona product of signed graphs exhibits cospectral properties, including L-cospectrality and Q-cospectrality. i have defined the corona product of two signed graphs and examined their structural and spectral properties. Based on this, I propose a signed network generative model by iteratively taking the corona product of a small graph with itself. This small graph, referred to as the seed graph, forms the basis for the resulting signed graphs, known as corona graphs. I have also derived fundamental properties of corona graphs, such as the number of signed links, signed triangles, degree distribution, and algebraic conflict. Finally, I have demonstrated that by carefully selecting a seed graph, the corresponding corona graphs can preserve the properties of real signed networks.

The spectral properties of the signed corona product of signed graphs can be used to find the sufficient and necessary conditions for the signed graph to be controllable.

5.2 FUTURE WORK

I believe that the corona product framework can be extended to define the corona product for gain graphs and weighted graphs. Additionally, the generalized corona product of signed graphs can be expanded similarly to how it is defined for unsigned graphs. Future

work should focus on studying the controllability of signed graphs, particularly in the context of their corona product. This is crucial for understanding the dynamics of complex networks with signed interactions, which is important for applications ranging from social network analysis to biological systems, where the nature of the interactions plays a significant role in the overall behavior and controllability of the network.