

Solidification and Shrinkage: Analytical and Numerical Model Development with Case Studies

A Thesis submitted by
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in partial fulfillment of the requirements for the award of the degree of
Doctor of Philosophy



॥ त्वं ज्ञानमयो विज्ञानमयोऽसि ॥

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Declaration

I hereby declare that the work presented in this thesis titled "*Solidification and Shrinkage: Analytical and Numerical Model Development with Case Studies*" submitted to the Indian Institute of Technology Jodhpur in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy, is a bonafide record of the research work carried out under the supervision of *Dr. Prodyut Ranjan Chakraborty*. The contents of this thesis in full or in parts, have not been submitted to, and will not be submitted by me to any other Institute or University in India or abroad for the award of any degree or diploma.

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Certificate

This is to certify that the thesis titled "*Solidification and Shrinkage: Analytical and Numerical Model Development with Case Studies*", submitted by *Aniket Dilip Monde (P15ME201)* to the Indian Institute of Technology Jodhpur for the award of the degree of *Doctor of Philosophy*, is a bonafide record of the research work done by him under my supervision. To the best of my knowledge, the contents of this report, in full or in parts, have not been submitted to any other Institute or University in India or abroad for the award of any degree or diploma.

Dr. Prodyut Ranjan Chakraborty
Ph.D. Thesis Supervisor

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Nomenclature

Variables	Description
C	concentration
C_e	eutectic concentration
c_p	specific heat (J/kgK)
D	diffusivity (m^2/s)
d	diameter (m)
F_o	Fourier number
f	mass fraction
g	volume fraction
g_a	gravitational acceleration (m/s^2)
H	height (m)
h	enthalpy (J/kg)
h_{sl}	latent heat (J/kg)
k	thermal conductivity (W/mK)
k_p	partition coefficient
L	domain length (m)
l	length (m)
m	mass (kg)
Pr	prandtl number
p	pressure (Pa)
T	temperature (K or $^{\circ}C$)
t	time (s)
T_c	cold boundary temperature (K or $^{\circ}C$)
T_e	eutectic temperature (K or $^{\circ}C$)
T_L	liquidus temperature (K or $^{\circ}C$)
T_m	melting temperature (K or $^{\circ}C$)
T_S	solidus temperature (K or $^{\circ}C$)
u, v, w	magnitude of velocity in x, y, and z directions, respectively (mm/s)
\vec{V}	velocity vector (m/s)
v	specific volume (m^3/kg)
v_i	crystal growth rate (m/s)
W	width (m)
x, y, z	coordinate axes
x_i	solid-liquid interface

Greek	Description
β	solidification contraction
β_T	thermal expansion coefficient
β_C	solubility expansion coefficient
ε	constant
ρ	density (kg/m^3)
μ	dynamic viscosity (kg/ms)
γ	surface energy (J/m^2)
α	thermal diffusivity (m^2/s)
λ	relaxation factor

Subscripts	Description
C	cavity
$i, 0$	initial
f	final
int	interface
l	liquid
R	riser
ref	reference value
v	void
s	solid

Superscripts	Description
k	present time-step
$k - 1, 0$	previous time-step
n	iteration step

List of Abbreviation

Abbreviation	Full form
<i>2D</i>	Two-Dimensional
<i>3D</i>	Three-Dimensional
<i>S/L</i>	solid-liquid
<i>ALE</i>	arbitrary Lagrangian-Eulerian
<i>CCD</i>	charge coupled device
<i>HRS</i>	high rate solidification
<i>LMC</i>	liquid-metal cooling
<i>LUDES</i>	linear upwind difference scheme
<i>ND</i>	non-dimensional
<i>NEE</i>	non-equilibrium eutectic
<i>PCM</i>	phase change materials
<i>PFT</i>	pseudo-front tracking
<i>PIV</i>	particle image velocimetry
<i>REV</i>	representative elementary volumes
<i>RMS</i>	root mean square
<i>SIMPLE</i>	semi-implicit method for pressure linked equations
<i>SIMPLER</i>	semi-implicit method for pressure linked equations revised
<i>TDMA</i>	tridiagonal matrix algorithm
<i>TNT</i>	trinitrotoluene
<i>VOF</i>	volume of fluid
<i>VGf</i>	vertical gradient freeze

