Acknowledgments

I am indebted to my Mathematics teachers during my schooling at *JNV Pokhal, Tehri Garhwal*, who made Mathematics a curious subject and made my basics strong. I am gifted with vast friend circle across *JNV Pokhal, IIT Jodhpur, IIT Kanpur, IIT Kharagpur, IIT Delhi, ISI Delhi* for making my life beautiful in spite of the several hurdles during my Ph.D.

I thank my thesis supervisor *Dr. Bibhas Adhikari* who was always more than just thesis supervisor. His teachings made me a mature person both academically and socially. You made it possible for me to work with various faculty members from premier Institutes of India. My gratitude is extended to *Dr. Vivek Vijay* for guiding me during the crucial phases of the thesis.

One of the superlative things of my life happened in July 2016 when I met Mathematician, *Professor RB Bapat* during his short visit to *IIT Kharagpur*. Since then he is constantly guiding me with his supreme knowledge. During my course work in initial days of Ph.D., I was in awe of him due to one of his sublime books *Graphs and Matrices*. It was a dream to work with him and it feels superb to accomplish that.

I am grateful to *Pradumn Pandey* who motivated me to do Ph.D. and without his indispensable support, it would have been not possible. I am deeply appreciative of *Anoopa Mishra, Amanjot Kaur, Rohan Sharma, Parmod Paul, Rakesh kumar* for being with me during my ups and downs. My gratitude is extended to *Supriyo Dutta* for the sound discussions and the reading of my research papers and thesis manuscript.

Being graduated from an IIT, you along with your parents, carry an immense pressure of expectation from the society. I acknowledge and thank my family, for the patience and love they ushered on me, and for bearing with me even when I was not spending less time with them. I pay respects to my *parents* for all their *love, sacrifices and blessings*…

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List of Symbols

Symbol	*Description*
G	Signed graph or Signed network
$A(G)$	Adjacency matrix of G
Cl_i	i-th cluster
C_n	Signed cycle graph on n vertices
$\det(A)$	Determinant of a square matrix A
$E(G)$	Edge set of G
$\phi(A)$	Characteristic polynomial of A
$\lambda_j(G)$	j-th eigenvalue of G
$K_n^{m,r}$	K_n with m negative cliques each of order r
K_n	Complete graph on n vertices with positive edges
L	Linear subdigraph of $A(G)$
mK_n	Mix complete graph on n vertices
$\|G\|$	Underlying positive graph of G
E_-	Negative edge set of G
E_+	Positive edge set of G
P_n	Signed path graph on n vertices
$\mathrm{per}(A)$	Permanent of a square matrix A
$\rho(G)$	Spectral radius of G
$\psi(A)$	Permanent polynomial of A
T_n	A signed tree on n vertices
$V(G)$	Vertex set of G
$w(L)$	Weight of L

List of Abbreviations

Abbreviation	*Full Form*
BCW	Balanced Weighted Closed Walk
UCW	Unbalanced Weighted Closed Walk