

## List of Abbreviation

Abbreviation	Full form
<i>2D</i>	Two-Dimensional
<i>3D</i>	Three-Dimensional
<i>ACS</i>	Adaptive Center Selection
<i>ANCPB</i>	Average Number of Checked Pixels per Block
<i>ARPS</i>	Adaptive Rood Pattern Search
<i>BC</i>	Block Classification
<i>BD</i>	Boundary
<i>BD – PSNR</i>	Bjontegaard Delta Peak-Signal-to-Noise-Ratio
<i>BD – Bitrate</i>	Bjontegaard Delta Bit-rate
<i>BFB</i>	Background Foreground Boundary
<i>BFDS</i>	Background-Foreground-Division-based Search
<i>BG</i>	Background
<i>BMA</i>	Block Matching Algorithm
<i>BP</i>	Block Partitioning
<i>bps</i>	Bits per second
<i>CDF</i>	Cumulative Distribution Function
<i>CDHS</i>	Cross Diamond Hexagon-shaped Search
<i>CDS</i>	Cross Diamond Search
<i>CR</i>	Compression Ratio
<i>CS</i>	Cross Search
<i>DCT</i>	Discrete Cosine Transform
<i>DI</i>	Dynamic Indexing
<i>DISC</i>	Dynamically chosen Initial Search Center
<i>DS</i>	Diamond Search
<i>DWT</i>	Discrete Wavelet Transform
<i>EDOS</i>	Efficient Direction-Oriented Search
<i>EGC</i>	Exponential Golomb Codes
<i>FAME</i>	Fast Adaptive Motion Estimation
<i>FG</i>	Foreground
<i>FPGA</i>	Field-Programmable Gate Array
<i>FS</i>	Full Search
<i>FSS</i>	Four-Step Search
<i>GPU</i>	Graphics Processing Unit
<i>HCI</i>	Human-Computer Interaction
<i>HEVC</i>	High-Efficiency Video Coding
<i>HS</i>	Hexagon-shaped Search
<i>HWDS</i>	Horizontal Wings Diamond Search
<i>IHS</i>	Inclined Hexagon-shaped Search
<i>IMV</i>	Intermediate Motion Vector
<i>JCT – VC</i>	Joint Collaborative Team on Video Coding
<i>JPEG</i>	Joint Photographic Experts Group
<i>KCDS</i>	Kite Cross Diamond Search

Abbreviation	Full form
<i>LCSP</i>	Large Cross Search Pattern
<i>LDSP</i>	Large Diamond Search Pattern
<i>LHSP</i>	Large Hexagon-shaped Search Pattern
<i>MAD</i>	Mean of Absolute Differences
<i>MHGS</i>	Modified Hexagon Grid Search
<i>MoCap</i>	Motion Capture
<i>MPEG</i>	Moving Picture Experts Group
<i>MSE</i>	Mean Squared Error
<i>MV</i>	Motion Vector
<i>NTSS</i>	New Three-Step Search
<i>OB</i>	Overlapping Blocks
<i>PCA</i>	Principal Component Analysis
<i>PDF</i>	Probability Distribution Function
<i>pSAD</i>	Partial SAD
<i>PSNR</i>	Peak-Signal-to-Noise-Ratio
<i>QP</i>	Quantization Parameter
<i>RD</i>	Rate-Distortion
<i>RMV</i>	Relative Motion Vector
<i>RoI</i>	Region-of-Interest
<i>SAD</i>	Sum of Absolute Differences
<i>SCSP</i>	Small Cross Search Pattern
<i>SDSP</i>	Small Diamond Search Pattern
<i>SHSP</i>	Small Hexagon-shaped Search Pattern
<i>SP</i>	Skeleton Prediction
<i>SPE</i>	Skeleton Prediction Error
<i>SR</i>	Search Region
<i>SS</i>	Search Strategy
<i>SSIM</i>	Structural Similarity Index
<i>TZS</i>	Test Zone Search
<i>TSS</i>	Three Step Search
<i>VLSI</i>	Very-Large-Scale Integration
<i>VMAF</i>	Video Multi-Method Assessment Fusion
<i>VWDS</i>	Vertical Wings Diamond Search
<i>WMSE</i>	Weighted Mean Squared Error
<i>WPSNR</i>	Weighted Peak-Signal-to-Noise-Ratio

## Declaration

I hereby declare that the work presented in this Thesis titled “*Efficient Motion Estimation and Predictive Coding Methods for Compression of Spatio-temporal Sequences*” submitted to the Indian Institute of Technology Jodhpur in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy, is a bonafide record of the research work carried out under the supervision of *Dr. Anil Kumar Tiwari*. The contents of this Thesis in full or in parts, have not been submitted to, and will not be submitted by me to, any other Institute or University in India or abroad for the award of any degree or diploma.



Tushar Shankar Shinde  
P15VSS003



## Certificate

This is to certify that the Thesis titled “*Efficient Motion Estimation and Predictive Coding Methods for Compression of Spatio-temporal Sequences*”, submitted by *Tushar Shankar Shinde (P15VSS003)* to the Indian Institute of Technology Jodhpur for the award of the degree of *Doctor of Philosophy*, is a bonafide record of the research work done by him under my supervision. To the best of my knowledge, the contents of this report, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

*Dr. Anil Kumar Tiwari*  
*Ph.D.Thesis Supervisor*



## Acknowledgments

I am thankful to God for the excellent health, courage, and strength that were necessary to complete this Thesis. I feel proud to thank my family, who taught me many great things in simple ways that helped me to survive the ups and downs of my life.

I owe my deepest gratitude and respect to my Ph.D. Thesis Supervisor, *Dr. Anil Kumar Tiwari*, for always encouraging me to explore the research area independently. In the process, I have learned many technical and non-technical aspects of professional work. I am grateful to him for his *time, patience, excellent guidance, and invaluable support* as well as for continually reminding me to be perfect in the little things that I do each day.

I am also very grateful to *Prof. Weiyao Lin*, for his *guidance, ideas, enlightening discussions, and excellent suggestions* during my research internship at Shanghai Jiao Tong University, Shanghai, China. Also, I am indebted to my doctoral committee members, *Dr. Gaurav Harit, Dr. Gaurav Bhatnagar, Dr. Chiranjoy Chattopadhyay, and Dr. Rajlaxmi Chouhan*, for their valuable comments, suggestions, and continued guidance during the research work.

Further, I am thankful to the Director of IIT Jodhpur for providing such excellent facilities and making my stay at the Institute comfortable and fruitful. I would also like to thank the Department of Electrical Engineering for providing the research facilities. I am also thankful to the faculty members of the department who were always supporting and helping. Also, I would like to thank all the staff members of the department for providing all sorts of support during my studies. I am grateful to the Office of Students for nominating me to represent the Institute at various international forums and student exchanges. I am also thankful to the Office of Academics for their continuous help and support. I would also like to thank Counselling Services and *Dr. Ankita Sharma* for giving me a chance to lead the team and help our Institute to be a home away from home. I am grateful to *Dr. Ankita Sharma* also for her motivational support and belief in me. I am grateful to the Ministry of Electronics and Information Technology (MeitY), Government of India, for initiating the visionary Visvesvaraya Ph.D. Scheme for Electronics and IT. The financial support provided through this scheme is gratefully acknowledged. I am also indebted to the Ministry of Youth Affairs and Sports, Government of India, for selecting me to be a part of an Indian youth delegation to Russia in 2017.

I wish to thank my lab-mates, *Aditya Raj, Deepak, Kumar Rahul, Naveen Mangal, Onkar Krishna, Puneet Jain, and Shabari Nath* for all their help. My stay at the Institute was a wonderful experience because of my friends, *Abhishek Sahu, Alankar Agarwal, Amrik Singh, Ankit Agarwal, Jyoti Faujdar, Pradumn Pandey, Rohit Kumar, Shivam Chaturvedi, and Shraddha Choudhary*. I wish to thank them all for the liveliness they infused into the non-academic part of the days at IIT Jodhpur. I would also like to thank and acknowledge *Ashwini*, who has been a constant source of strength, love, and inspiration.

Finally, I thank all my well-wishers who have directly or indirectly contributed to my work. I am grateful to my late grandmother. Even though she did not have formal education, she never stopped sharing her wisdom, tutelage, support, and encouragement to progress. I acknowledge and thank my family, especially my parents, my sister, *Pooja*, and my brother, *Kishor*, for their relentless belief in me. I owe my deepest gratitude to my father and mother, who have devoted their everything to our education with invariant love and enthusiasm. They have made tremendous sacrifices to survive this family in tough days. I could never have gone this far without every bit of their faith.

*Tushar Shankar Shinde*  
Ph.D. Student



# List of Figures

Figure	Title	page
1.1	(a) Illustration of typical Posetrack skeleton sequence, (b) bit-rate for video and skeleton content in Posetrack, (c) typical surveillance skeleton sequence, (d) bit-rate for video and skeleton content in surveillance. The skeleton sequences are compressed by the traditional fixed-length direct coding approach (left bins) and our proposed skeleton coding approach (right bins). (Best viewed in color)	5
2.1	Illustration of a differential encoding scheme.	10
2.2	Illustration of a lossy encoding and decoding scheme.	12
2.3	Illustration of a video encoding process.	13
2.4	Traditional block matching process for $M \times N$ candidate block.	14
3.1	Average MV distribution characteristics for slow and medium motion video sequences: (a) Average MV distribution for slow motion video sequences, (b) Average MV distribution for slow motion video sequences after dynamically selecting SR based on location of DISC, (c) CDF of MV distribution for slow motion video sequences, (d) Average MV distribution for medium motion video sequences, (e) Average MV distribution for medium motion video sequences after dynamically selecting SR based on location of DISC, (f) CDF of MV distribution for medium motion video sequences.	23
3.2	Average MV distribution characteristics for fast and directional motion video sequences: (a) Average MV distribution for fast motion video sequences, (b) Average MV distribution for fast motion video sequences after dynamically selecting SR based on location of DISC, (c) CDF of MV distribution for fast motion video sequences, (d) Average MV distribution for directional motion video sequences, (e) Average MV distribution for directional motion video sequences after dynamically selecting SR based on location of DISC, (f) CDF of MV distribution for directional motion video sequences.	24
3.3	Illustration of proposed block matching algorithm with dynamically switched SR for candidate block corresponding to the minimum SAD location: $(0, 0)$ or $(x_m, y_m)$ .	25
3.4	Search patterns: (a) 1 <sup>st</sup> step pattern: SDSP, (b) 2 <sup>nd</sup> step north directional pattern, (c) 2 <sup>nd</sup> step east directional pattern, (d) 2 <sup>nd</sup> step south directional pattern, (e) 2 <sup>nd</sup> step west directional pattern, (f) 3 <sup>rd</sup> step onward horizontal directional pattern: HWDS, (g) 3 <sup>rd</sup> step onward vertical directional pattern: VWDS, (h) 3 <sup>rd</sup> step onward inclined directional pattern: $+45^\circ$ IHS, (i) 3 <sup>rd</sup> step onward inclined directional pattern: $-45^\circ$ IHS.	27
3.5	Selection of one of the four direction-oriented search patterns based on the coordinate location $(\Delta x_i, \Delta y_i)$ .	28
3.6	Typical search path examples: (a) 1 <sup>st</sup> step search stop with predicted MV $(0, 0)$ , (b) 3 <sup>rd</sup> step search stop with predicted MV $(1, 0)$ , (c) 3 <sup>rd</sup> step search stop with predicted MV $(1, 1)$ , (d) 4 <sup>th</sup> step search stop with predicted MV $(2, 2)$ , (e) 8 <sup>th</sup> step search stop with predicted MV $(5, 4)$ . Note: The checked search points are marked with the corresponding step number. The IMV and predicted MV is represented in the gray and dark circle respectively.	28
3.7	Ratio of SADs obtained with sub-sampling and without sub-sampling and denoted by $\phi$ , follows Gaussian distribution.	29
3.8	Representative frames of the test video sequences used in the analysis.	31

3.9	Performance comparison in terms of PSNR and speed-up for proposed method against different block matching algorithms: (a) PSNR comparison for different algorithms without sub-sampling, (b) Speed-up comparison for different algorithms without sub-sampling, (c) PSNR comparison for different algorithms with 1 : 2 sub-sampling, (d) Speed-up comparison for different algorithms with 1 : 2 sub-sampling, (e) PSNR comparison for different algorithms with 1 : 4 sub-sampling, (f) Speed-up comparison for different algorithms with 1 : 4 sub-sampling.	35
3.10	(a) PSNR comparison for different variants of proposed method against FS, (b) Speed-up comparison for different variants of proposed method against FS.	36
3.11	Rate-distortion performance for video sequences (a) Discussion (slow motion), (b) Coastguard (medium motion), (c) Soccer (fast motion), and (d) Parkjoy (directional motion) under different QP values of 22, 27, 32, and 37.	36
4.1	(a) Illustration of traditional block classification approach with two classes: background and foreground, (b) proposed approach with three classes: background, foreground, and boundary, (c) two classes example (square sequence), (d) three classes example (square sequence). (Best viewed in color)	40
4.2	Statistics of MV distribution: (a) traditional BG-FG approach, (b) Our BG-FG-boundary approach, statistics of average MAD distribution: (c) traditional BG-FG approach, (d) Our BG-FG-boundary approach.	41
4.3	Framework of the proposed background foreground boundary aware motion search scheme.	42
4.4	Block classification framework.	43
4.5	Different block overlap scenarios for block classification.	45
4.6	Search patterns: (a) 1 <sup>st</sup> step pattern: SDSP, (b) 2 <sup>nd</sup> step east directional pattern, (c) 2 <sup>nd</sup> step south directional pattern, (d) 2 <sup>nd</sup> step west directional pattern, (e) 2 <sup>nd</sup> step north directional pattern, (f-m) 3 <sup>rd</sup> step onward search patterns: (f) 0° directional pattern: RUWDS, (g) -45°, (h) -90°, (i) -135°, (j) 180°, (k) 135°, (l) 90°, and (m) 45°.	46
4.7	(a) Illustration of coordinate location $(\Delta x_i, \Delta y_i)$ . One of the eight direction-oriented search patterns is chosen based on $(\Delta x_i, \Delta y_i)$ , (b) Typical search path example. The corresponding step number is marked in all the traversed search points.	47
4.8	(a) 16 × 16 block (pixel grid) with region-based 1 : 4 sub-sampling. Highlighted pixels of five different regions are used for partial distortion computation, (b) Illustration of 16 × 16 current block (highlighted in green) and corresponding four overlap blocks (highlighted in red).	48
4.9	BD block partitioning process. The size of circle is proportional to the MAD values. BG and FG sub-regions are highlighted in blue and red color respectively.	49
4.10	Different BD block partitioning scenarios: (a) single region R4, (b) two adjacent regions R4 and R3, (c) opposite regions R4 and R2, (d) R4 and central region Ro, (e) R4, R3 and Ro, (f) R4, R1, R3 and Ro. BG and FG regions are highlighted in blue and red color respectively.	49
4.11	Representative frames of surveillance sequences used in the analysis.	51
4.12	The effect of variation in fixed and adaptive thresholds with different multiplying factor $\beta$ for block classification. Column 1-4 considers the initial-threshold value as 256, 512, 1024, 2048, respectively.	53
4.13	Results of the proposed block classification method. (a) Original frame, Campus (46 <sup>th</sup> frame), (d) Original frame, Crossroad (6 <sup>th</sup> frame), (g) Original frame, Square (50 <sup>th</sup> frame), (b), (e), and (h) 2-Class classification, (c), (f), and (i) Our 3-Class classification.	55
4.14	Rate-distortion performance for video sequences (a) Ice, (b) Square, (c) Bank, and (d) Pedestrian area under different QP values of 22, 27, 32, and 37.	57
4.15	Block-class-wise PSNR (in dB) comparison for different algorithms against FS. Row 1-4 corresponds to different sequences: Hall monitor (type-A), Square (type-B), Bank (type-C), and Mainroad (type-D), respectively. (Best viewed in color)	59

4.16	Block-class-wise Speed-up (in times) comparison for different algorithms against FS. Row 1-4 corresponds to different sequences: Hall monitor (type-A), Square (type-B), Bank (type-C), and Mainroad (type-D), respectively. (Best viewed in color)	60
5.1	Overview of proposed skeleton coding method.	64
5.2	Example of skeleton representation in a frame.	66
5.3	Illustration of spatial differential skeleton coding method. (The direction arrow indicates the parent-child relationship)	67
5.4	Illustration of MV-based skeleton coding method: (a) skeletons in the $(t - 1)^{th}$ reference frame and current $(t^{th})$ frame, (b) demonstration of MV computation, and (c) demonstration of skeleton alignment and MV-based skeleton prediction. (Best viewed in color)	69
5.5	Illustration of Trajectory-based skeleton coding method: (a) skeletons in the $(t - 2)^{th}$ , $(t - 1)^{th}$ reference frames, and current $(t^{th})$ frame, (b) trajectory difference between two reference frames highlighted in red arrow, and (c) skeleton trajectory prediction highlighted in red arrow and skeleton prediction error highlighted by yellow arrow. (Best viewed in color)	69
5.6	Illustration of Relative MV (RMV)-based skeleton coding method: (a) MV-based skeleton prediction method, (b) RMV-based skeleton encoding for body joint 12 (right-ankle), the comparison of prediction MV-based and RMV-based prediction error is highlighted in circle. (Best viewed in color)	70
5.7	Adaptive prediction mode selection based on the causal skeleton body joint mode information. (Best viewed in color)	72
5.8	(a) Illustration of prediction residual computation for the current frame and the reference frame, (b) Illustration of difference vector computation. (Best viewed in color)	74
5.9	Illustration of adaptive center selection based on the length of current error vector and difference error vector: (a) $(r_x, r_y)$ is considered as adaptive center, and (b) $(0, 0)$ is considered as adaptive center. The adaptive center is shown in yellow circle.	74
5.10	(a) Illustration of prediction residual in reference frames, (b) symbol frequency lookup table creation based on prediction residual in reference frames, (c) illustration of prediction residual in current frame, (d) mapping current prediction residual to the lookup table, and (d) generating index stream for current frame prediction residual symbols. A more frequent residual value is assigned with a smaller index number. (Best viewed in color)	76
5.11	Entropy mode switching scheme.	77
5.12	Example skeleton sequences: (a) Posetrack sequence- Ice skating, (b) Posetrack sequence- Volleyball, (c) Surveillance sequence- Night, and (d) Surveillance sequence- Footpath.	79
5.13	Comparison between the overall version of our approach and the direct coding & temporal differential coding methods. Our approach includes all of the prediction, entropy coding, and side information coding schemes proposed in this chapter. The direct coding method use direct coding to code side information. The temporal differential coding method use EGC to encode prediction residuals and temporal differential coding to encode side information.	85
5.14	Example wild skeleton sequences: (a) Ice skating, (b) Volleyball.	85



## List of Tables

Table	Title	page
2.1	Overview of popular block matching algorithms (BMA).	15
3.1	Test video sequences used in the study.	30
3.2	Performance comparisons in terms of PSNR, SSIM, and ANCPB for slow motion sequences.	32
3.3	Performance comparisons in terms of PSNR, SSIM, and ANCPB for medium motion sequences.	32
3.4	Performance comparisons in terms of PSNR, SSIM, and ANCPB for fast motion sequences.	33
3.5	Performance comparisons in terms of PSNR, SSIM, and ANCPB for directional motion sequences.	33
4.1	Surveillance test sequences used in the study.	51
4.2	PSNR and motion search complexity (ANCPB) comparisons for different components of proposed method. The full version of proposed method contains all three components: 3-class block classification (BC), class-based search strategy (SS), and block partitioning (BP).	58
4.3	PSNR and motion search complexity (ANCPB) comparisons for different traditional and surveillance-based block matching algorithms.	58
5.1	Test skeleton sequences used in the study.	77
5.2	Bits per skeleton points and Bit-savings for different prediction modes (EGC is used to encode all residuals and the bits for coding side information are excluded).	81
5.3	Statistics of bits per body joint and bit-savings for different entropy coding schemes (the adaptive multimodal prediction method is used as the prediction mode).	83
5.4	Bits per skeleton points and Bit-savings for different prediction modes and different entropy coding schemes. (The bits for coding side information are excluded)	83
5.5	Skeleton side information coding performance comparison for different methods (in bits per skeleton joint and bit savings).	84
5.6	Statistics of bits per joint & bit-savings for different frame skip scenarios on Posetrack & Surveillance datasets.	86
5.7	Statistics of bits per joint & bit-savings for skeletons estimated [Xiu et al., 2018] on Posetrack & Surveillance datasets.	86
5.8	Statistics of bits per joint & bit-savings for different Gaussian noise levels on Posetrack & Surveillance datasets.	86



## List of Symbols

Symbol	Description
$x_i$	$i^{th}$ symbol in the data source
$P(x_i)$	Probability of occurrence of $i^{th}$ symbol in the data source
$d_i$	Difference between two consecutive symbols in the data source
$N$	Length of data sequence
$(x, y)$	Position of the candidate block
$M \times N$	Size of the candidate block
$(m_x, m_y)$	Motion vector of the candidate block
$p$	Search displacement parameter
$n$	Number of intermediate search steps
$T$	Pre-set threshold value for SAD
$(x_m, y_m)$	Median of MVs of adjacent blocks
$(p_x, p_y)$	Maximum fixed SR dimension in horizontal and vertical directions
$(p_x^d, p_y^d)$	Maximum adaptive displacement in horizontal and vertical directions
$R_0$	SR corresponding to SR center $(0, 0)$
$R_m$	SR corresponding to SR center $(x_m, y_m)$
$(\Delta x_i, \Delta y_i)$	Coordinate difference of IMVs of the $(i - 1)^{th}$ and $(i - 2)^{th}$ search steps
$\beta$	Pixel sub-sampling parameter
$\varphi$	A ratio of SADs obtained with sub-sampling and without sub-sampling
$T_{1:\beta}^{new}$	New threshold value where $1 : \beta$ is the sub-sampling
$P_i(MV)$	The probability distribution of MV
$ANCPB_{1:\beta}$	ANCPB for $1 : \beta$ sub-sampled SAD
$ANCPB_{FS}$	ANCPB for FS algorithm
$ANCPB_{fast\_algorithm}$	ANCPB for fast block matching algorithm
$V_{max}$	Maximum pixel intensity for the given bit resolution
$\alpha$	The proportion of pixels used in the sub-sampled SAD computation
$pfbg$	Background region in the previous frame
$n_{pfbg}$	Total number of BG blocks in the previous frame
$\overline{SAD}_{pfbg}$	Adaptive threshold based on SADs for background region in the previous frame
$S$	Class number (0: BG, 1: BD, 2: FG)
$\omega_s$	Weight corresponding to class $S$
$ C_s $	Total number of blocks belonging to class $S$
$MSE(m, n)$	Mean-squared-error of $(m, n)^{th}$ candidate block
$j_i = (x_i, y_i)$	The horizontal and vertical coordinates of the body joint $j_i$
$N_J$	Total number of body joints
$O_i$	Occlusion information for the body joint $j_i$
$SID_s^t$	Skeleton ID of the $s^{th}$ skeleton in the $t^{th}$ frame
$O_s^t$	Occlusion information of the $s^{th}$ skeleton in the $t^{th}$ frame
$O_{s,i}^t$	Occlusion flag corresponding to the $i^{th}$ body joint of the $s^{th}$ skeleton in the $t^{th}$ frame
$J_s^t$	body joint coordinate information of the $s^{th}$ skeleton in the $t^{th}$ frame
$N_S^t$	Number of skeletons present in the $t^{th}$ frame

Symbol	Description
$F^t$	Complete skeleton information in the $t^{th}$ frame
$j_i^p$	Parent body joint for the body joint $j_i$
$w_{j_i^p}$	Weight corresponding to the $j_i$ 's parent body joint $j_i^p$
$w_f$	Weight corresponding to the $(t - f)^{th}$ reference frame
$N_f$	Number of temporal frames used for bit-requirement estimation
$(c_x, c_y)$	Current prediction residual
$(r_x, r_y)$	Reference prediction residual
$(d_x, d_y)$	Difference prediction residual
$b_{SID_s^t}^t$	Number of bits used to encode skeleton ID ( $SID_s^t$ )
$b_{O_s^t}^t$	Number of bits used to encode the occlusion information corresponding to the $s^{th}$ skeleton in the $t^{th}$ frame
$\widehat{b}_{mode}^t(j_i)$	Estimated the bit-requirement for $j^{th}$ body joint in the selected prediction mode
$b_{total}^t$	Total number of bits required to encode complete skeleton information corresponding to the $t^{th}$ frame
$b_{N_s^t}^t$	Total number of bits required to encode $N_s^t$
$b_{j_i}^t$	Total number of bits required to encode $j^{th}$ body joint