

# On Some Important Reliability Aspects of General Coherent Systems

A Thesis submitted by

**Tanmay Sahoo**

In partial fulfilment of the requirements for the award of the degree of

**Doctor of Philosophy**



॥ त्वं ज्ञानमयो विज्ञानमयोऽसि ॥

**Indian Institute of Technology Jodhpur**

**Department of Mathematics**

**November 2023**



**Dedicated To  
My Beloved Grandmothers**

*“Late Giri Bala Sahoo and Late Narmada Sahoo”*



# Declaration

I hereby declare that the work presented in this thesis entitled “**On some important reliability aspects of general coherent systems**” submitted to the Indian Institute of Technology Jodhpur in partial fulfillment of the requirements for the award of the degree of Doctor of Philosophy, is a bonafide record of the research work carried out under the supervision of **Dr. Nil Kamal Hazra**. The contents of this thesis in full or in parts, have not been submitted to, and will not be submitted by me to, any other Institute or University in India or abroad for the award of any degree or diploma.

  
Tanmay Sahoo

P19MA001



# Certificate

This is to certify that the thesis entitled “**On some important reliability aspects of general coherent systems**” being submitted by **Tanmay Sahoo** to the Indian Institute of Technology Jodhpur, India, for the award of degree of Doctor of Philosophy in Science is a record of bonafide research work carried out by him under my supervision during the last four years. The results embodied in this thesis, to the best of my knowledge, have not been submitted for the award of any other degree/diploma anywhere. In my opinion, the thesis fulfils the requirement for the award of the degree.

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# List of Notation

$X$	underlying absolutely continuous nonnegative random variable
$\mathbf{X}$	random vector
$f_X(\cdot)$	probability density function of $X$
$F_X(\cdot)$	cumulative distribution function of $X$
$F_X^{-1}(\cdot)$	quantile function of $X$
$\bar{F}_X(\cdot)$	survival (reliability) function of $X$
$E(X)$	expectation (mean) of $X$
$r_X(\cdot)$	hazard (failure) rate function of $X$
$\tilde{r}_X(\cdot)$	reversed hazard (failure) rate function of $X$
$m_X(\cdot)$	mean remaining (residual) life function of $X$
$\Delta_X(\cdot)$	$-\ln \bar{F}_X(\cdot)$
$\tilde{\Delta}_X(\cdot)$	$-\ln F_X(\cdot)$
$\mathcal{C}(\cdot)$	copula function
$\bar{\mathcal{C}}(\cdot)$	survival copula function
$\phi(\cdot)$	generator of Archimedean copula
$R(u)$	$u\phi'(u)/\phi(u)$
$H(u)$	$u\phi'(u)/(1-\phi(u))$
$G(u)$	$u\phi''(u)/\phi'(u)$
$\psi(\cdot)$	inverse of the generator $\phi$ of an Archimedean copula
$exp(\cdot)$	exponential function
$X_t$	$(X - t   X > t)$
$\mathcal{N}$	the set of natural numbers
$\mathbb{R}$	the set of real numbers
$\mathbb{R}_+$	the set of positive real numbers
$A^n$	$n$ cartesian product of the set $A$
increasing	non-decreasing
decreasing	non-increasing
positive	non-negative
negative	non-positive
$a \stackrel{\text{def.}}{=} b$	$a$ is defined as $b$

$u'(t)$	the first derivatives of $u(t)$ with respect to $t$
$u''(t)$	the second derivatives of $u(t)$ with respect to $t$
$u^{(k)}(t)$	the $k$ -th derivatives of $u(t)$ with respect to $t$
$\stackrel{d}{=}$	equality in distribution
$g^{-1}(\cdot)$	inverse function of $g(\cdot)$
$I(\cdot)$	indicator function
$E_k$	collection of all $n$ dimensional points such that $k$ coordinates are one and $(n - k)$ coordinates are zero
$A^c$	complement of the set $A$
$ A $	cardinality of the set $A$
$T$	lifetime of a coherent system
$\tau$	structure function of a coherent system
$P_i$	minimal path set of a coherent system
$\mathcal{A}$	set of all minimal path sets of a coherent system
$\tau_{r n}$	structure function of an $r$ -out-of- $n$ system
$h_T(\cdot)$	reliability function of a coherent system with lifetime $T$
$h_{r:n}(\cdot)$	reliability function of an $r$ -out-of- $n$ system
$h_T(\mathbf{p})$	$h_T(p_1, \dots, p_n)$ , $0 < p_i < 1$ , for all $i = 1, \dots, n$
$h_{r n}(\mathbf{p})$	$h_{r n}(p_1, \dots, p_n)$ , $0 < p_i < 1$ , for all $i = 1, \dots, n$
$h_T(p)$	$h_T(\mathbf{p})$ whenever $p_i = p$ , for all $i = 1, \dots, n$
$h_{r n}(p)$	$h_{r n}(\mathbf{p})$ whenever $p_i = p$ , for all $i = 1, \dots, n$
$E_X(\cdot)$	empirical distribution function of $X$
$\bar{E}_X(\cdot)$	empirical reliability function of $X$

# List of Abbreviations

Abbreviation	Full form
st	usual stochastic
hr	hazard rate
rh	reversed hazard rate
lr	likelihood ratio
mrl	mean remaining lifetime
disp	dispersive
icx	increasing convex
icv	increasing concave
PDF	probability density function
CDF	cumulative distribution function
RF	reliability function
IID	independent and identically distributed
DID	dependent and identically distributed
DNID	dependent and non-identically distributed
ILR	increasing likelihood ratio
DLR	decreasing likelihood ratio
IFR	increasing failure rate
DFR	decreasing failure rate
IFRA	increasing in failure rate average
MIFRA	multivariate increasing failure rate in average
DFRA	decreasing in failure rate average
NBU	new better than used
NWU	new worse than used

OS	ordinary order statistics
SOS	sequential order statistics
GOS	generalized order statistics
DSOS	developed sequential order statistics
DGOS	developed generalized order statistics
FGM	Farlie-Gumbel- Morgenstern copula
CO	Clayton-Oakes copula
PHR	proportional hazard rate
WCS	weighted coherent system
UGF	universal generating function
MRI	Birnbaum marginal reliability importance
JRI	Birnbaum joint reliability importance
BP	Barlow-Proschan lifetime importance
SWI	structure-based weighted importance