

Geometric Tolerance Estimation Framework

4.1 INTRODUCTION

The manufactured components deviate in size, form, and orientation in comparison to the features envisaged by the designer due to various process faults. The major process faults that lead to the deviation of manufactured components during the end milling operation are deflections of cutting tool and workpiece, tool wear and breakage, cutter runout, fixturing errors, etc. The static deflections of cutting tool and workpiece are the primary source for the deviation of machined components from the design specifications during end milling of thin-walled components. These deviations are expressed in the form of geometric tolerances as per the Geometric Dimensioning and Tolerancing (*GD&T*) principles [ASME Y14.5-2009, 2009] or [ISO 1101:2017, 2017] using size, form, and orientation of the features. According to *GD&T* principles, the geometric tolerances of straight and constant curvature components are designated by flatness and cylindricity respectively. This chapter presents a computational framework to estimate static deflection induced geometric tolerances during end milling of thin-walled straight and constant curvature or circular components. The overall framework to estimate the geometric tolerances require systematic integration of several computational models. The framework includes models to predict cutting forces, estimate coordinates representing distorted machined surface due to tool and workpiece deflections, and mechanism to transform distorted coordinates into geometric tolerances for straight and circular thin-walled components. The mechanistic model discussed in the previous chapter has been implemented to estimate cutting forces during milling of straight and circular components. The prediction of distorted machined coordinates requires deflection models for cutting tool and workpiece. The present study adopts cantilever beam formulation and an FE-based models to estimate tool and workpiece deflections respectively. The distorted coordinates are extracted in the form of point-cloud data from both these models, and the same is transformed into a geometric tolerance using the PSO algorithm. The individual elements of the proposed framework are implemented in the form of synchronized computational models, and a set of machining experiments are conducted over a wide range of cutting conditions to substantiate the proposed framework. This chapter summarizes the development of an overall framework for the estimation of flatness and cylindricity related parameters during end milling of thin-walled components.

The thesis work also aims at analyzing the relative contributions of tool and workpiece deflections on the geometric tolerances with the variation of cutting parameters and workpiece geometry. Subsequently, the thesis analyzed some of the inevitable aspects associated with the end milling of thin-walled circular components, such as concave-convex side machining and workpiece rigidity. It is accomplished by estimating geometric tolerances for different geometries while migrating from zero curvature to constant curvature geometries. Further, the thesis work investigates the effect of structural attributes related to curved components such as change in the radius of curvature, variation in radial engagement area and chip load while machining from convex and concave side. The effect of change in the radial engagement area due to workpiece curvature has been investigated by introducing the concept of “*Equivalent Radial Depth of Cut*”. Whereas, the effect of variation in the radius of curvature has been investigated by varying workpiece curvature keeping other structural parameters and machining

conditions unchanged for concave and convex geometries. The effect of workpiece rigidity during machining of thin-walled components is an important parameter which is also addressed in this chapter. It is demonstrated that an appropriate combination of productive cutting conditions and the component thickness could improve geometric tolerances considerably. The computational framework proposed in the present study can assist process planners in selecting appropriate cutting conditions to manufacture thin-walled components within tolerance limits specified by the designer.

4.2 ESTIMATION OF DISTORTED MACHINED SURFACE

The thin-walled components are inherently flexible and prone to significant deflections under the application of periodically varying cutting forces during the end milling operation. Similarly, the end mill is another flexible element of the milling system that also deflects considerably under the action of cutting forces. The static deflections of cutting tool and workpiece contribute significantly towards distorted machined surface and violation of geometric tolerances. Therefore, it is desirable to estimate these deflections to estimate dimensional and geometric inaccuracies of machined components. The flexibility of the thin-walled component increases further with the progress of milling operation due to the material removal. The increase in the workpiece flexibility has significant effect on static deflections of the component and thereby generation of distorted machined surface. The static deflections of the thin-walled component lead to variation of the cutting forces during the end milling operation [Wan et al., 2005], [Wan and Zhang, 2006a]. It is important to incorporate these aspects in the modeling of distorted surface and thereby the geometric tolerances.

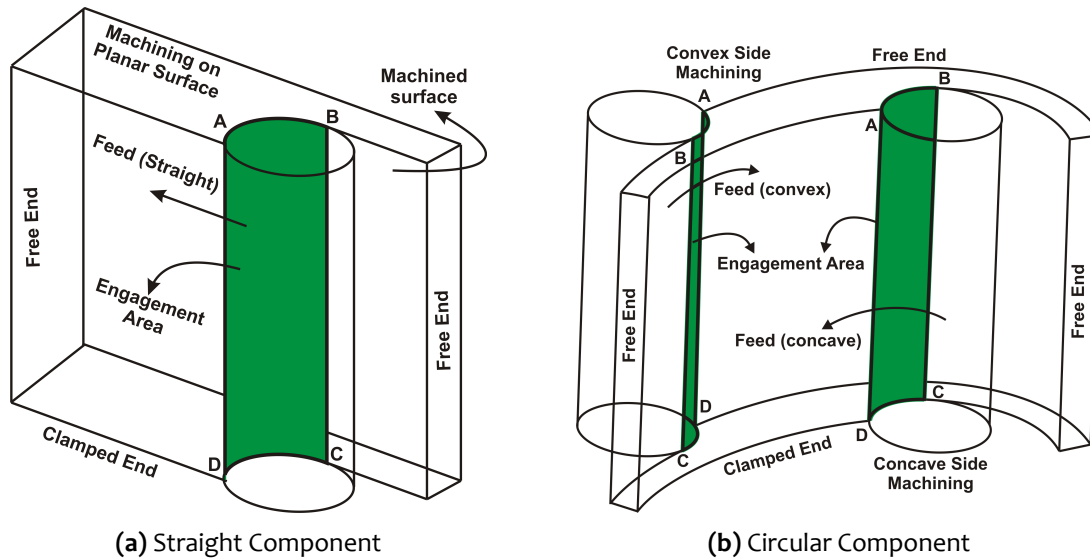


Figure 4.1 : Tool-workpiece Transition Area and Boundary Conditions

Thin-walled components with curved surfaces exhibit significantly different deflection characteristics compared to planar or straight components as depicted in Figure 4.1. The circular components offer the choice of machining from either the concave or convex side which has a considerable effect on machining characteristics of a component. The engagement area is considerably larger while machining from the concave side meanwhile it is much lesser for the convex side in comparison to straight geometries under similar cutting conditions. The change of engagement area alters the engagement pattern of cutting flutes, cutting force, and thereby error in the form of geometric tolerances for straight (flatness) and circular (cylindricity) components.

The 3-Dimensional (3-D) FE Analysis is employed in the present study to estimate deflections of thin-walled components using ANSYS Parametric Design Language (APDL) environment integrated with the MATLAB 2019b [2019]. The Cantilever beam model with non-uniform application of cutting forces is employed to predict static deflections of an end mill [Budak and Altintas, 1995].

4.2.1 Tool Deflection Model

The tool deflection model considers end mill or cutting tool as a Cantilever beam with non-uniform forces acting on the cutter-workpiece engagement area. The model is quite realistic, generic and easy to integrate with the mechanistic force model outlined in the previous chapter. The model uses the concept of equivalent diameter (D_e) to incorporate the effect of cutting flutes while estimating deflections of an end mill [Kops and Vo, 1990]. The equivalent diameter represents a solid cylinder without helical flutes but resulting in the identical deflections as an actual cutter exhibits under similar cutting conditions. The methodology divides an end mill into a number (n) of axial disc elements along the axial direction, as shown in Figure 4.2.

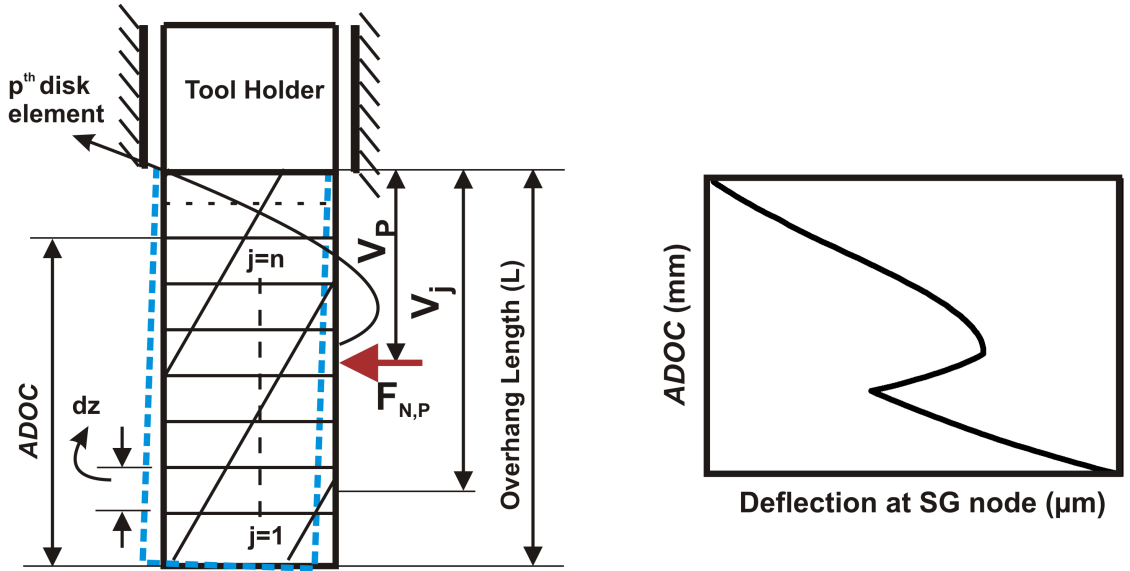


Figure 4.2 : Schematic Illustration of Tool Deflection Model

The thickness of axial disc elements in both, cutting force and tool deflection model is maintained at the same level to enable the direct transfer of cutting forces. The Cantilever beam formulation estimates the deflection of individual axial disc element (j) in the normal $\delta_N(j, p)$ direction due to cutting force acting on p^{th} ($1, 2, \dots, n$) element using Eqs. 4.1 and 4.2.

$$\delta_N(j, p) = \frac{F_{N,p} v_j^2}{6EI} (3v_p - v_j) \quad 0 < v_j < v_p \quad (4.1)$$

$$\delta_N(j, p) = \frac{F_{N,p} v_p^2}{6EI} (3v_j - v_p) \quad v_p < v_j \quad (4.2)$$

In Eqs. 4.1 and 4.2, v_p is determined as $v_p = L - (dz)(p)$. The area moment of inertia ($I = \frac{\pi D_e^4}{64}$) for the end mill is calculated using the equivalent diameter of the cutter ($D_e = s(2R_c)$), where s is a scale factor due to helical flutes for the end mill [Kops and Vo, 1990]. The total static deflection

in the normal direction $\delta_N(j)$ at a given axial node (j) can be calculated by the superposition of deflections produced by forces acting on all in-cut nodes using Eq. 4.3.

$$\delta_N(j) = \sum_{p=1}^n \delta_N(j, p) \quad (4.3)$$

4.2.2 Workpiece Deflection Model

The static deflections of straight and circular thin-walled components are modeled in the present study using 3-D Finite Element (FE) based methodology owing to the ease of its integration with the mechanistic force model. The 3-D FE modeling is necessary to predict static deflections of thin-walled components as workpiece rigidity diminishes significantly due of reduction of component thickness with the progress of machining. Although 3-D modeling is computationally expensive, it can effectively mimic interactions between tool-workpiece during the milling operation. The component is meshed using a 3-D 8-node solid shell element (SOLSH 190) to ensure the advantages of using solid elements and avoid shear locking [Wang, 2006]. Figure 4.3 depicts meshed models of thin-walled components showing straight and circular geometries studied in this thesis work.

The thin-walled component has been modeled and analyzed considering Free-Free-Free-Clamped conditions, which is common in the end milling operation. The component is free to deflect along three sides i.e. top, right and left while it is constrained from the bottom as shown in Figure 4.3a. The interaction zone of cutting tool and workpiece at any instant is termed as transition zone and it is marked as ABCD in Figure 4.3a. The flute enters the cutting zone from vertex D and along edge DA while it exits along edge CB with vertex B traversed last. The expanded form of the transition zone and movement of flute over the zone at different instants is depicted using the slant line inclined at a helix angle (θ_h) in Figure 4.3a. The cutting forces estimated using mechanistic model outlined in the previous chapter are transferred to the transition zone of meshed FE model and applied along the slant line representing the helical cutting edge of the end mill as represented in Figure 4.3b. In order to facilitate the direct transfer of elemental cutting forces to nodes of the FE model, the mesh parameters are selected in a systematic manner. The width (d) of a quad element is selected such that the aspect ratio is equal to $\tan(\theta_h)$ as shown in Figure 4.3a. The height (h) of the quad element is identical as thickness of the disk element (dz) in a cutting force model. The other portion is meshed accordingly by sweeping the area mesh along thickness of the component. For each incremental rotation of the cutter, forces computed for individual disk element are applied as depicted in Figure 4.3b and nodal deflections are computed.

The deflection of each node in the transition zone is not important as surface generation occurs only when a flute passes through the exit edge CB. The exit edge CB is termed as surface generation line, and corresponding nodes lying on the edge CB are termed as Surface Generation (SG) nodes as depicted in Figure 4.3c. The surface generation commences when a flute passes through the SG node lying at point C and terminates when it finally traverses vertex B. The elemental cutting forces are applied to FE model of the workpiece and nodal deflections are estimated as discussed above. The static deflections in the tangential $\delta_T(j)$ and normal $\delta_N(j)$ direction for j^{th} SG node at a given instant are recorded along with its actual coordinates $acor_c$ ($c = x, y, z$) to represent distorted machined surface. Subsequently, the cutter is rotated such that the SG point slides by amount equal to the thickness of axial disc elements in the cutting force model. The elemental cutting forces corresponding to the new location of a flute are computed and applied to the FE model for determining workpiece deflections. These steps are repeated at each surface generation instant until deflection is computed for each SG node along line CB. Once deflection

values are computed, it has to be transformed in the form of coordinates representing the distorted machined surface. The subsequent section summarizes the methodology to accomplish the task of obtaining deflected coordinates from deflection models.

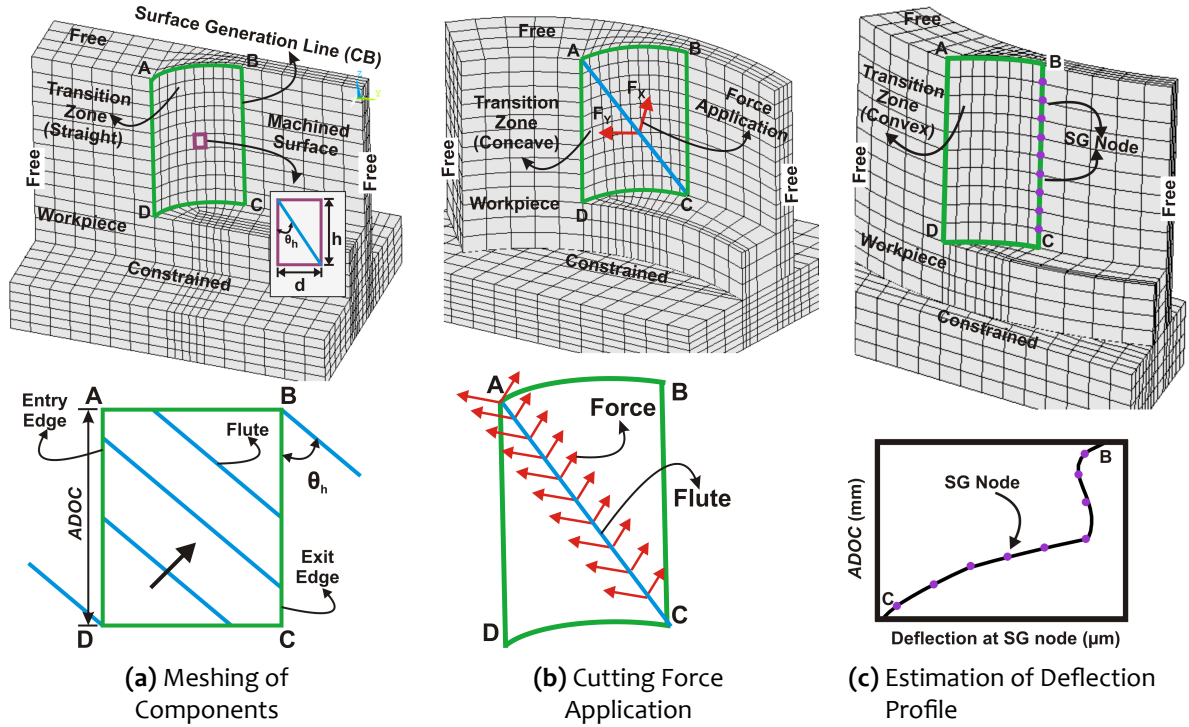


Figure 4.3 : Schematic Illustration of Workpiece Deflection Model

A Global Coordinate System (GCS) is defined at the center of the circular component for the application of cutting forces available from the mechanistic model. A “moving” coordinate system in the form of a Local Coordinate System (LCS) has been used in the FE analysis to compute deflection values in the tangential (δ_T) and normal (δ_N) directions at each instant of surface generation as depicted in Figure 4.4. Also, the rigidity of thin-walled components diminishes considerably with the progress of milling operation due to removal of material. The reduction of workpiece rigidity has significant effect on static deflections of the component and magnitude of workpiece deflection varies along the length of the component in the feed direction. Therefore, it is important to incorporate these aspects in the computational model and investigate its effects on distorted machined surface and geometric tolerances. In order to appreciate this aspect, static deflections are estimated at L different locations ($L = 1, 2, 3, \dots, q$) along the feed direction as shown in Figure 4.4 for the concave side machining. The similar procedure has been employed for the straight and convex side machining.

The cutting forces in the end milling operation are dependent on cutting parameters e.g. $ADOC$, $RDOC$, feed rate etc. The $RDOC$ is altered significantly in the presence of tool-workpiece deflections and actual machining operation occurs at a much lower value than the programmed or actual $RDOC$ ($RDOC_{act}$). The reduced $RDOC$ changes the magnitude of cutting force and thereby influences deflections and generation of distorted machined surface. Therefore, it is necessary to incorporate this aspect while estimating cutting forces and tool-workpiece deflections. The same has been achieved by implementing an iterative procedure that uses the deflection data obtained previously to calculate corrected $RDOC$ value ($RDOC_{rev}$) using Eq. 4.4. The $RDOC_{rev}$ is used subsequently to estimate the revised cutting forces. The deflection values obtained using tool and workpiece deflection models are added to derive the data in the form of discrete values $\delta(L, r, j)$

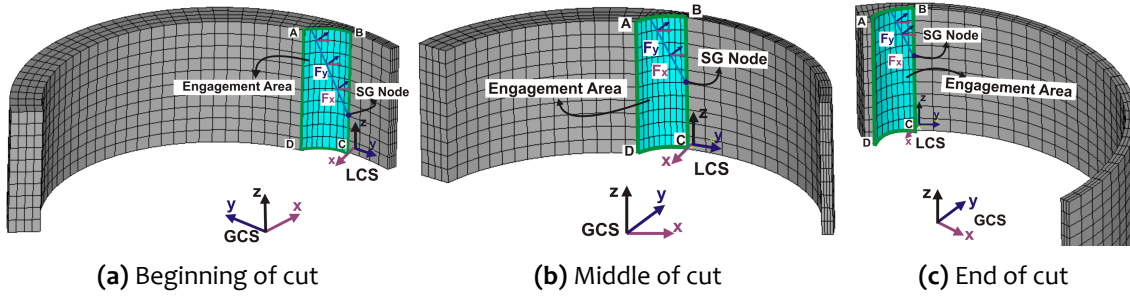


Figure 4.4 : FE Analysis of Thin-walled Components with the Progress of Material Removal

for the axial disk element (j) at L^{th} position along the feed direction and r^{th} iteration. The discrete values $\delta(L, r, j)$ for each axial disk element (j) need to be simplified in the form of a single value $a_d(L, r)$ for the correction of $RDOC$. The simplified value is obtained by computing the arithmetic mean for each axial disk element (j) at the individual L^{th} position along feed direction for r^{th} iteration using Eq. 4.5. The $RDOC_{rev}$ obtained using Eq. 4.4 is used in the subsequent iteration ($r + 1$) to predict cutting forces. The process is repeated iteratively until the change of $RDOC$ converges to a stable value expressed using Eq. 4.6. The variation of $RDOC_{rev}$ as a function of iterations (r) is shown in Figure 4.5 . It can be observed that the $RDOC_{rev}$ converges to a stable value after a few iterations (r) and it is considerably lower than $RDOC_{act}$ yielding reduced cutting forces and enhanced rigidity for the thin-walled components. The process mentioned above is implemented computationally to accomplish the entire task in an automated manner at L different locations along the direction of feed to estimate tool-workpiece deflections.

$$RDOC_{rev}(L, r) = RDOC_{act} - a_d(L, r) \quad (4.4)$$

$$a_d(L, r) = \frac{\sum_{j=1}^n \delta(L, r, j)}{n} \quad (4.5)$$

$$|RDOC_{rev}(L, r) - RDOC_{rev}(L, r - 1)| < 0.01mm \quad (4.6)$$

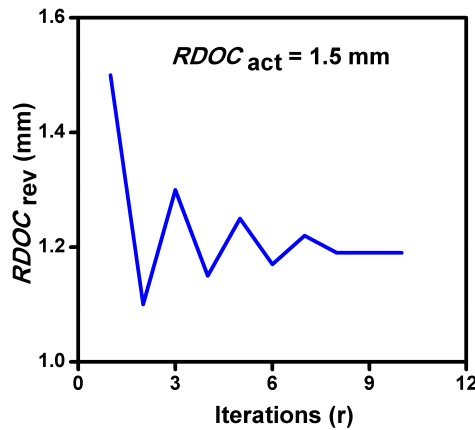


Figure 4.5 : Variation of Corrected $RDOC$

4.2.3 Estimation of Deflected Coordinates

The static deflections of an end mill and thin-walled component require further transformations in the form of point-cloud data representing distorted machined surface to estimate geometric tolerances. The transformation necessitates synchronization among the local and global coordinate systems. The total deflection value $def_c^{LCS}(L, j)$, ($c = x, y, z$) computed at each (j^{th}) SG node is subtracted from the coordinate $acor_c^{LCS}(L, j)$, ($c = x, y, z$) representing ideal surface in the LCS to obtain distorted coordinate $dcor_c^{LCS}(L, j)$, ($c = x, y, z$) using Eq. 4.7. It is necessary to transform the distorted coordinates computed in LCS into GCS located at the center of the workpiece using Eq. 4.8.

$$dcor_c^{LCS}(L, j) = acor_c^{LCS}(L, j) - def_c^{LCS}(L, j) \quad (c = x, y, z) \quad (4.7)$$

$$\begin{bmatrix} dcor_x(L, j) \\ dcor_y(L, j) \\ dcor_z(L, j) \end{bmatrix}^{GCS} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dcor_x(L, j) \\ dcor_y(L, j) \\ dcor_z(L, j) \end{bmatrix}^{LCS} + \begin{bmatrix} R_f \cos \theta \\ R_f \sin \theta \\ 0 \end{bmatrix} \quad (4.8)$$

Figure 4.6 presents the overall framework to determine distorted coordinates representing machined surface for the straight and circular thin-walled components. The methodology can be divided into three stages broadly; the first stage consists of integrating the mechanistic force model with the tool and workpiece deflection models. The second stage deals with the estimation of tool-workpiece deflections. The third stage deals with the transformation of deflection data into the point cloud. An automated program is developed in this study to compute point cloud representing machined surface during milling of thin-walled components.

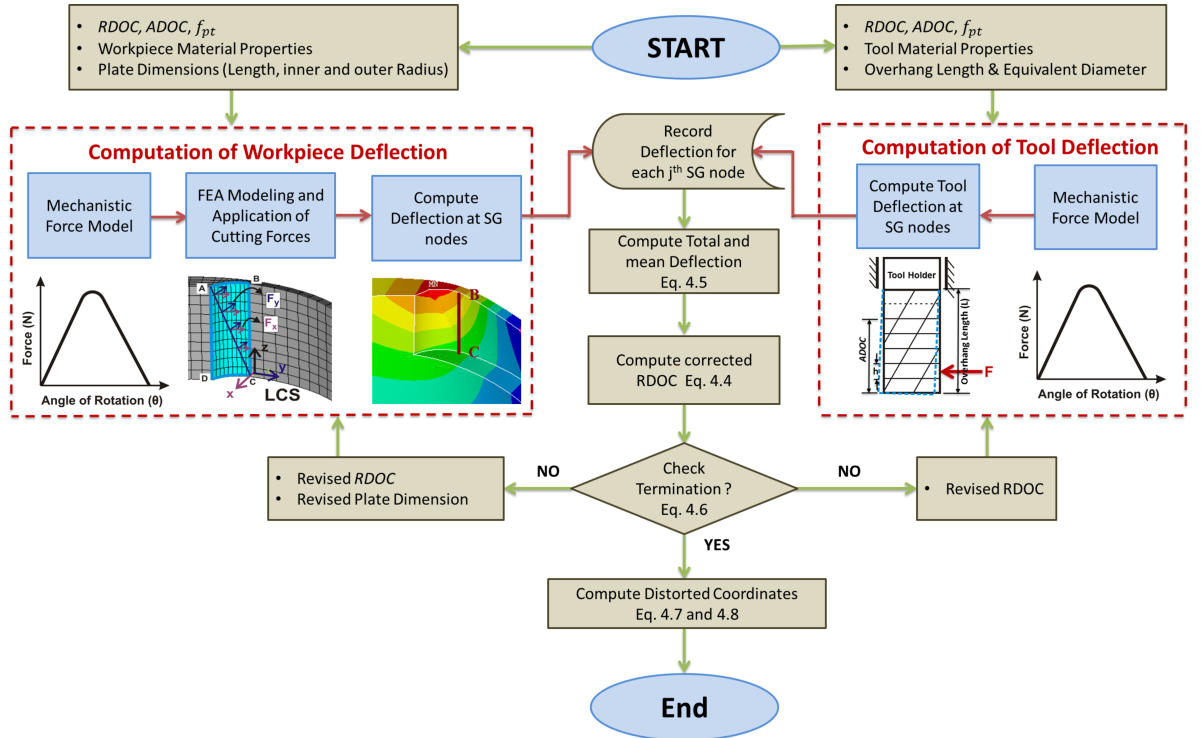


Figure 4.6 : Overall Framework for Estimation of Distorted Coordinates

The coordinates obtained in this manner imitates the distorted machined surface representing thin-walled components. The previous studies reported in the literature obtain distorted coordinates from *CMM* or other measurement systems without any linkages with the process faults such as workpiece deflections. However, the present study estimates point-cloud data using physics-based models considering process faults that aid process planners in deciding appropriate machining conditions to meet geometric tolerance requirements. The framework to estimate process-induced geometric tolerances forms another fundamental contribution of the present thesis work. The subsequent section presents the PSO-based methodology to estimate geometric tolerances from the distorted coordinates representing the machined surface.

4.3 EVALUATION OF GEOMETRIC TOLERANCES

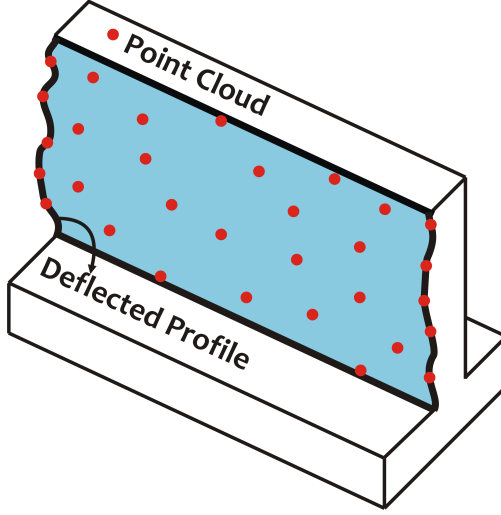
Geometric Tolerance is a measure of the deviation for manufactured products from an ideal geometric feature in terms of size, form and orientation. The size represents feature dimensions and the form is defined as an orthogonal distance between the two parallel bounding features encompassing point-cloud representing the distorted machined surface. The orientation denotes the inclination of normal to the feature or axis of the feature. The present study aims at determination of geometric tolerances for straight and circular components therefore flatness and cylindricity are under consideration. The present study employs numerical technique based on genetic algorithm namely, Particle Swarm Optimization (PSO) for the estimation of geometric tolerances. The PSO technique was developed by Eberhart and Kennedy [1995] and it was employed in multiple applications owing to the ease of computational implementation with requirement of conditioning fewer parameters compared to other algorithms. It provides an accurate solution with faster convergence in multiple cases and preferred for various non-linear optimization problems [Kovvur et al., 2008]. The objective function for determination of flatness and cylindricity is developed as a non-linear minimum zone optimization problem in the present thesis work. The derivation of objective function depends on the feature reference parameters used for the optimization of the minimum normal distance between the nearest and the farthest point. The subsequent sections present the formulation of the objective function, followed by algorithm implementation in estimating flatness and cylindricity for straight and circular thin-walled components.

4.3.1 Objective Function for Flatness

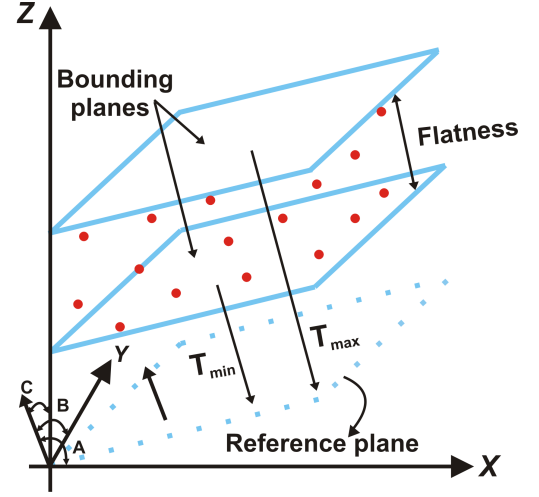
The reference geometric feature for the determination of flatness is a plane and it can be mathematically expressed using Eq. 4.9. The flatness of a component is designated using two attributes, (i) orthogonal distance between the two parallel bounding planes encompassing point-cloud representing distorted machined surface and, (ii) inclination of the normal to parallel planes. Figure 4.7 shows a simplified diagram highlighting elements used in the estimation of the flatness for the straight thin-walled components. The coordinates representing distorted machined surface $dcor_c(t)$ ($c = x, y, z$), ($t = 1, 2, \dots, n * q$) derived using models outlined in the previous section are used as an input to the algorithm. In Eq. 4.9, $\cos(Q)$ ($Q = A, B, C$) represent direction cosines of a normal to the reference plane while D is a constant representing intercept along Z - axis as depicted in Figure 4.7b. It is necessary to evaluate direction angles A, B, C and the constant D to derive flatness parameters of the feature. The present study confines a reference plane to pass through the origin, thereby yielding value of constant D as zero for the simplification. As sum of squares for direction cosines is unity, direction angle (C) corresponding to Z - component of the normal to a reference plane can be estimated using Eq. 4.10.

$$dcor_x \cos(A) + dcor_y \cos(B) + dcor_z \cos(C) - D = 0 \quad (4.9)$$

$$C = \cos^{-1}(\sqrt{1 - \cos^2(A) - \cos^2(B)}) \quad (4.10)$$



(a) Distorted Point-cloud due to Deflections



(b) Flatness and its Parameters

Figure 4.7 : Illustration of Geometric Tolerance Parameters for Straight Components

The algorithm requires evaluation of an independent set of parameters namely, direction angles (A, B) for the X- and Y- component of the normal to a reference plane for the flatness determination subsequently resulting into second order objective function. The independent resulting parameter vector (PV^f) with its range and objective function $Flat(PV^f)$ for the evaluation of flatness are defined using Eqs. 4.11 - 4.13.

$$PV^f = (A, B) \quad A, B \in [0, \pi] \quad (4.11)$$

$$Min\{Flat(PV^f) = Max(T^t) - Min(T^t)\} \quad (4.12)$$

$$T^t = dcor_x^t \cos(A^f) + dcor_y^t \cos(B^f) + dcor_z^t \cos(C^f) \quad (4.13)$$

4.3.2 Objective Function for Cylindricity

The reference geometric feature for the determination of cylindricity is a cylinder that can be parametrized using the position and orientation of the axis. The cylindricity is defined as the orthogonal distance between two minimum-zone co-axial cylinders enveloping the complete set of point-cloud data representing distorted machined surface. Figure 4.8 shows the schematic representation of cylindricity expressed by two co-axial cylinders encompassing the point-cloud data. The random point specifies the position of a point $P_a(x_a, y_a, z_a)$ on the axis and orientation is defined using direction cosines $Cos(Q)$ ($Q = A, B, C$) as depicted in Figure 4.8. The orthogonal distance (R^t) between the point-cloud data $P_t(dcor_x, dcor_y, dcor_z)$, ($t = 1, 2, \dots, n * q$) representing the thin-walled circular component and its axis can be given using Eq. 4.14.

$$R^t = |[Cos(A) Cos(B) Cos(C)] X [P_t - P_a]| \quad (4.14)$$

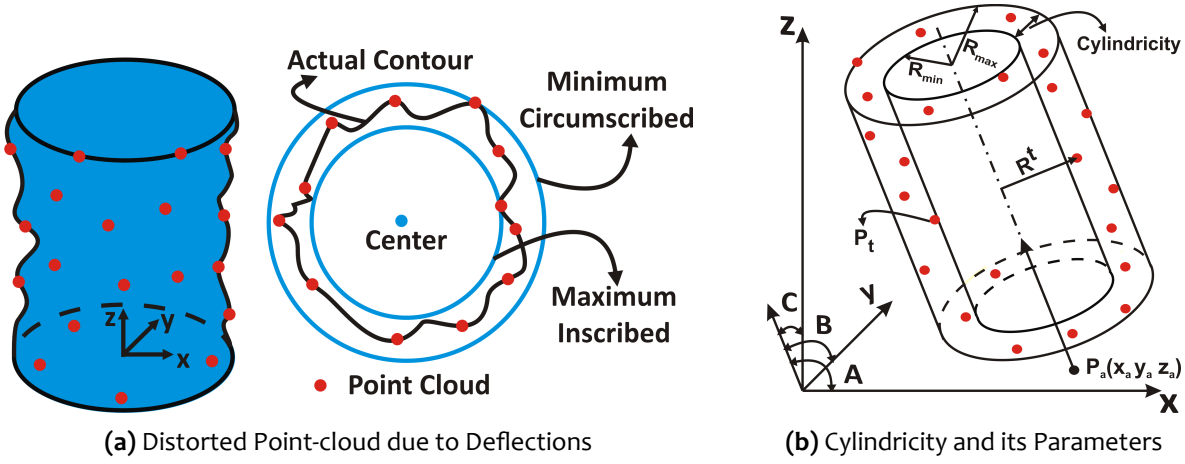


Figure 4.8 : Illustration of Geometric Tolerance Parameters for Circular Components

It is necessary to evaluate direction angles A, B, C and coordinates of the point $P_a(x_a, y_a, z_a)$ to compute cylindricity value. The present study confines point P_a on the cylinder axis within the XY - plane, therefore the Z - coordinate (z_a) can be considered as zero. The direction angle (C) corresponds to Z - component of the normal to a reference plane which can be estimated using Eq. 4.10. The algorithm requires an estimation of four independent parameters (PV^c) presented using Eq. 4.15 to determine the minimum cylindricity zone. It necessitates evaluation of the fourth-order objective function $Cycl(PV^c)$ to determine cylindricity defined using Eq. 4.16.

$$PV^c = [A, B, x_a, y_a] \quad (4.15)$$

$$Min\{Cycl(PV^c) = Max(R^t) - Min(R^t)\} \quad (4.16)$$

4.3.3 Algorithm Implementation

This section summarizes the implementation of the PSO algorithm in determining flatness and cylindricity parameters from distorted point-cloud data obtained for straight and circular geometries using computational models outlined in the previous section. The algorithm refers to particles as a set of random possible solutions termed as swarms that travel within the range of solution space with an adjusted velocity. A set of random solutions are initialized and a geometric tolerance value is evaluated for each particle using an objective function formulated as Eqs. 4.12 and 4.16. The particles in the algorithm store two entities; an individual set of parameters with the best geometric tolerance value attained till current iteration, and a set of parameters having the best geometric tolerance value obtained hitherto for any particle among the whole population. The optimal solution is determined by updating the position of a particle depending on the best geometric tolerance value obtained from a set of initialized solutions. The stepwise procedure of determining geometric tolerance using the PSO algorithm is summarized as follows;

1. Randomly generate particles (m), ($l = 1, 2, \dots, m$) with an initial guess of parameter vector (PV_l) and velocity vector (V_l) for each particle. The V_l assist particles in guiding from one position to another in each iteration. Define parameters relevant to PSO, e.g., boundary constraints (lower and upper bound of search space), number of iterations (u), ($1, 2, \dots, v$), population size (m), inertia weights (w_{max}, w_{min}), acceleration coefficients (c_1, c_2) and termination condition.

2. Evaluate geometric tolerance value using an objective function formulated in Eqs. 4.12 and 4.16 for each particle during the first iteration. For each l^{th} particle, the set of parameters (PV_l) are designated as individual local best set of parameters (LL_{best}). Whereas, the set of parameters for a particle with the lowest value of flatness (f_{best}) and cylindricity (c_{best}) are designated as global best set of parameters (GL_{best}) during the first iteration ($u = 1$).
3. The velocity of each particle and parameter set for the next iteration ($u + 1$) are updated according to its individual local best (LL_{best}) and global best (GL_{best}) set of parameter using Eqs. 4.17 - 4.19 respectively:

$$V_l(u + 1) = w(u) V_l(u) + c_1 r_1 (LL_{best}(u) - PV_l(u)) + c_2 r_2 (GL_{best}(u) - PV_l(u)) \quad (4.17)$$

$$PV_l(u + 1) = PV_l(u) + V_l(u + 1) \quad (4.18)$$

$$w(u) = (w_{max} - w_{min}) \frac{(v - u)}{u} + w_{min} \quad (4.19)$$

Here, PV_l = Current parameters of the particle; V_l = Current velocity of the particle; LL_{best} = Individual local best parameters of the particle; GL_{best} = Global best parameters of the particle; r_1, r_2 = Independently uniformly distributed random variables within range $[0, 1]$; w = Inertia weight; c_1, c_2 = Acceleration coefficients

4. Evaluate geometric tolerance i.e. flatness and cylindricity value using objective function formulated as Eqs. 4.12 and 4.16 for each particle. If current geometric tolerance value of any particle is lower than its previous value, then the individual best set of parameter (LL_{best}) is replaced by the current set of parameters. Similarly, if the best geometric tolerance value among the whole swarm for current iteration ($u + 1$) is lower than the global best value (f_{best}, c_{best}), the global best geometric tolerance value and set of parameter are replaced by current best geometric tolerance value and its associated set of parameters respectively.
5. The termination condition is checked and if it is not met, steps 3 onwards are repeated further. The termination criterion is the maximum number of iterations (v) in the present study.

Figure 4.9 summarizes the step-wise procedure to determine geometric tolerances. The effectiveness of the PSO algorithm in determining global minima depends on specific attributes such as the number of particles in a swarm, their initial positions, inertia weights, and acceleration coefficients. If the particles are too less, the algorithm may not investigate solutions space effectively and get trapped into a local minima. The large size of the swarm searches the solution space more effectively however it increases the computation time. In the present thesis work, the computation time was not a constraint; therefore, a sufficiently large number of particles were applied so that algorithm does not confine to a local minima. The initial position of particles also affects the performance of PSO algorithm. The random variables r_1 and r_2 are used for obtaining different sets of parameters for each particle scattered in the entire solution space. The values of variables are distributed randomly within the interval $[0, 1]$. The algorithm is executed repeatedly to nullify the effect of discrepancy in generating a random number and substantiate the optimal value of geometric tolerance and its parameters. The objective of inertia weight (w) in Eq. 4.17 is to govern the exploration competencies of a swarm which can be estimated using Eq. 4.19 for u^{th} iteration. If inertia weight is higher, the swarm flows to unexplored areas in the search space. If inertia weight is set lower, the capability of the swarm to flow is confined within the current search area. It is anticipated to have a linearly decreasing value of inertia weight with increment in iterations. The initial value of inertia weight is fixed as $w_{max} = 0.9$ in the present study which

reduces linearly to $w_{min} = 0.4$ over v iterations [Rezaee and Jasni, 2013]. The aim of acceleration coefficients c_1 and c_2 is to guide a particle to flow in the direction of its individual and global best set of parameters. If the value of an acceleration coefficient is higher, the particle flows abruptly and may get confined to local optimum geometric tolerance values. If the value is set lower, particles flow slowly with increased computational requirements. The value of c_1 and c_2 is set as 2, to have a balanced flow of particles in the present study.

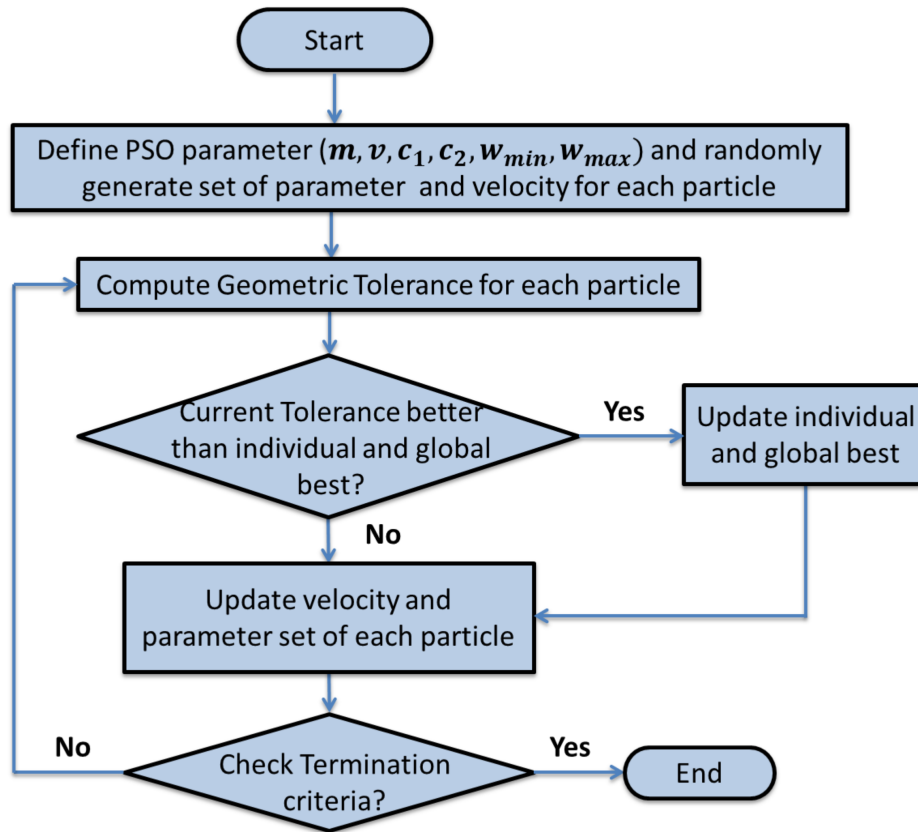


Figure 4.9 : Algorithm for Estimation of Geometric Tolerance using PSO

4.4 RESULTS AND DISCUSSION

The framework outlined in the previous section has been implemented using MATLAB 2019b [2019] and APDL platform to estimate geometric tolerances for the straight (flatness) and circular (cylindricity) thin-walled components. The cutting forces estimated through the mechanistic force model were input to cutter and workpiece deflection models to estimate distorted coordinates representing the machined surface. The distorted coordinates were transformed to the flatness and cylindricity parameters using PSO algorithm. The end milling experiments involving thin-walled components were conducted using solid carbide end mill and Aluminium 6061-T6 workpiece material. Tables 4.1 and 4.2 summarize the specification of cutting tool and attributes related to thin-walled planar and circular components (concave and convex geometries) applied during computational and experimental studies. The process parameters ($RDOC$, $ADOC$ and feed rate) and other cutting conditions are summarized in Table 4.3. The setups used during machining experiments (3-Axis Vertical milling machine, cutting tool, workpiece, etc.) and measurement of distorted coordinates (spindle mounted inspection probe) are depicted in Figures 4.10a and 4.10b respectively.

Table 4.1 : Tool Attributes

Tool Specification - Solid Carbide (Kennametal)						
Cutter Radius (R_c)	Helix Angle (θ_h)	Number of Flutes (n)	Scaling factor (s)	Equivalent Diameter $D_e = s(2 R_c)$	Overhang Length (L)	Young's Modulus (E)
8 mm	30°	4	0.75	12 mm	45 mm	550 GPa

Table 4.2 : Workpiece Attributes

Workpiece Specification - Aluminium 6061-T6					
Geometry	Radius (mm)		Workpiece Dimension (mm) (Length x Height x Thickness)	Young's Modulus	Poisson's Ratio
	Inner	Outer			
Straight	NA	NA	100 x 50 x 3.5	68.9 GPa	0.33
Concave	37.5	41			
Convex	39	42.5			
Straight	NA	NA	100 x 50 x 6		
Concave	35	41			
Convex	39	45			

Table 4.3 : Machining Conditions for Computational and Experimental Studies

Test No.	RDOC (mm)	ADOC (mm)	Final Plate Thickness (mm)	Engagement Angle (θ_{en})		
				Straight	Concave	Convex
1	1.5	4	2	35.66°	39.76°	32.84°
2		11				
3		18				
4		25				
5	4	4		60°	66.77°	55.86°
6		11				
7		18				
8		25				
Feed : 300 mm/min						
Spindle Speed : 2000 RPM						

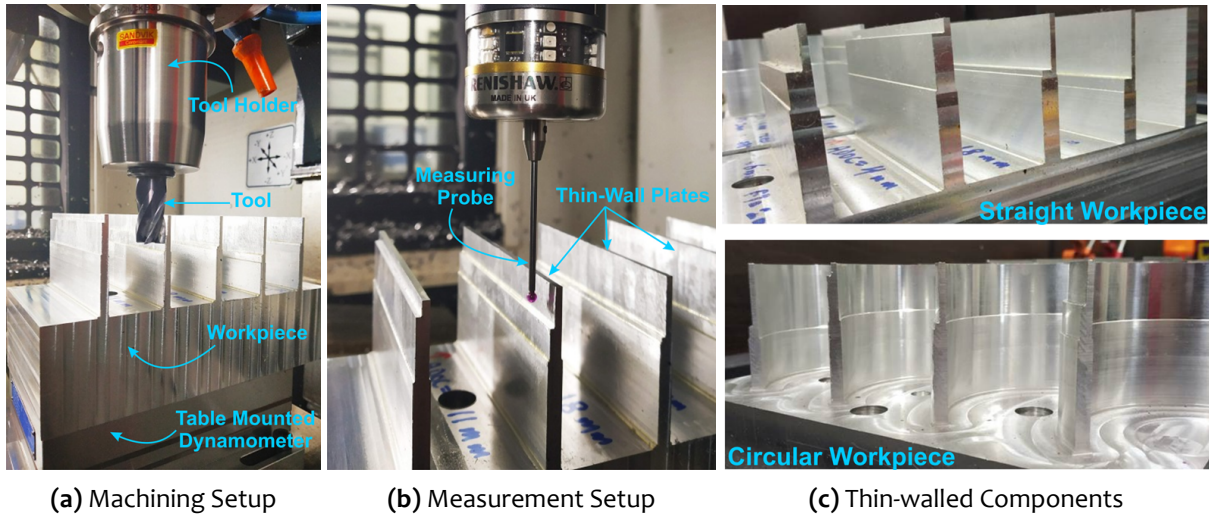


Figure 4.10 : Experimental Setup

4.4.1 Estimation of Geometric Tolerances

This section presents results obtained from the set of computational models presented in the previous section and machining experiments to examine the efficacy of the proposed framework in estimating the geometric tolerances during end milling of thin-walled straight, circular concave and convex components. The target thickness of thin-walled components after machining is kept constant at 2 mm for the studies to achieve the consistency and comparison of results. The *RDOC* and radius of curvature (for circular component) are varied to achieve an appropriate unmachined geometric configuration of the component. The *ADOC* is varied in the range of 4-25 mm to obtain various combinations of cutting widths. The coordinates representing the distorted machined surface are estimated at an interval of 10 and 1 mm along length and height of components respectively. The computationally estimated and experimentally measured coordinates are input to the PSO-based geometric tolerance determination algorithm for estimation of flatness and cylindricity parameters.

Table 4.4 : Computational and Experimental Flatness Values

Test No.	Computational (μm)		Experimental (μm)
	Workpiece Deflections	Tool-Workpiece Deflections	
1	40.720	40.720	52.623
2	191.935	191.935	167.159
3	201.825	202.450	183.872
4	288.046	292.073	262.417
5	8.525	26.428	34.332
6	58.224	68.626	67.259
7	77.433	83.553	85.056
8	149.394	157.830	161.519

Table 4.4 and 4.5 summarize flatness and cylindricity values obtained from computational models and machining experiments for cutting conditions summarized in Table 4.3. It can be seen that the geometric tolerance values estimated using proposed computational framework are in

good agreement with experimentally measured values for each case. The minor difference between computational and experimental results for few cases can be attributed to process disturbances such as geometric or kinematic errors of a machine tool, thermal errors, machining chatter, etc. that are not incorporated in the computational framework. It is observed that the geometric tolerance parameters have considerable dependence on cutting widths viz. *ADOC* and *RDOC*. The change of cutting widths affects static deflections of end mill and thin-walled components thereby altering the magnitude and distribution of distorted coordinates representing machined surface resulting into variation of geometric tolerances. It justifies the fundamental contribution of the present thesis work to develop a computational framework to estimate geometric tolerances during the end milling of thin-walled components. Figures 4.11 and 4.12 depicts the effectiveness of the proposed computational framework by comparing geometric tolerance and its parameters for straight, circular concave and convex side components corresponding to Test 2 and 6.

Table 4.5 : Computational and Experimental Cylindricity Values

Test No.	Concave			Convex		
	Computational (μm)		Experimental (μm)	Computational (μm)		Experimental (μm)
	Workpiece Deflections	Tool-Workpiece Deflections		Workpiece Deflections	Tool-Workpiece Deflections	
2	86.94	86.94	79.95	65.91	65.91	64.94
3	100.44	100.87	87.51	84.10	84.52	70.49
4	143.10	145.11	141.27	126.13	127.15	122.94
6	34.62	36.75	42.30	23.30	26.05	40.70
7	32.66	43.77	56.27	28.10	39.15	52.46
8	74.61	88.33	103.36	78.21	83.16	91.82

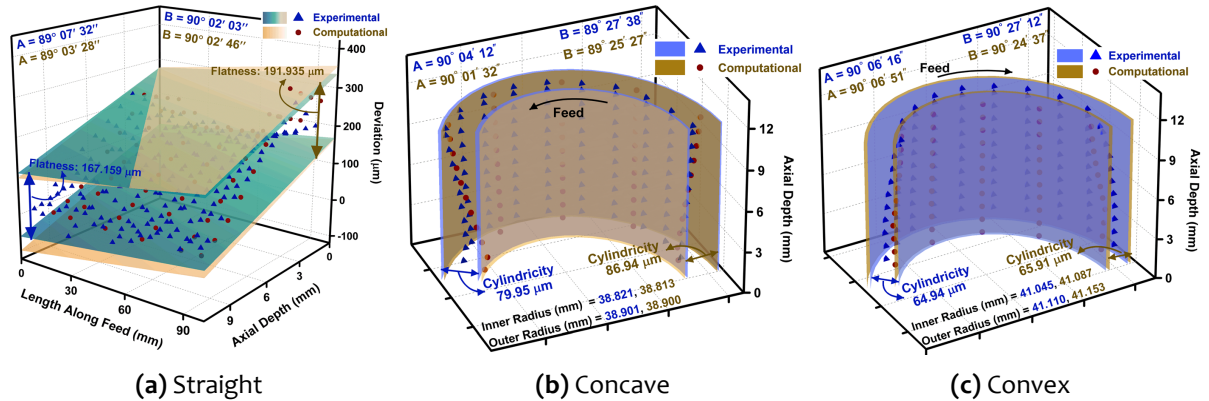


Figure 4.11 : Comparison of Geometric Tolerance Variation for Straight and Circular Geometries
Test 2 - *RDOC* = 1.5 mm; *ADOC* = 11 mm

The present thesis work estimates geometric tolerances using two distinct approaches; the first approach neglects tool deflections while the second approach uses both tool and workpiece deflections in predicting geometric tolerance. This study helps in appreciating the relative contributions of tool and workpiece deflections during milling of thin-walled straight and circular components. The relative difference of geometric tolerance among both approaches indicates the contribution of tool deflection in the geometric tolerance. It can be seen that the approaches with and without tool deflections estimate identical geometric tolerance values for cutting combinations having lower *RDOC* values (Test 1-4). The identical results can be attributed to the lower rigidity of the semi-finished/unmachined component that increases workpiece deflections remarkably.

Therefore, the inclusion of tool deflections does not alter estimated values considerably as workpiece deflection is a major contributor to geometric tolerances. The other set of cutting conditions having higher *RDOC* (tests 5-8), represents cutting conditions with higher rigidity of unmachined component accompanied by elevated cutting forces due to productive cutting conditions. The increased cutting forces results into increased contribution of tool deflections. This is further accompanied by reduced workpiece deflections due to enhanced rigidity of the unmachined component.

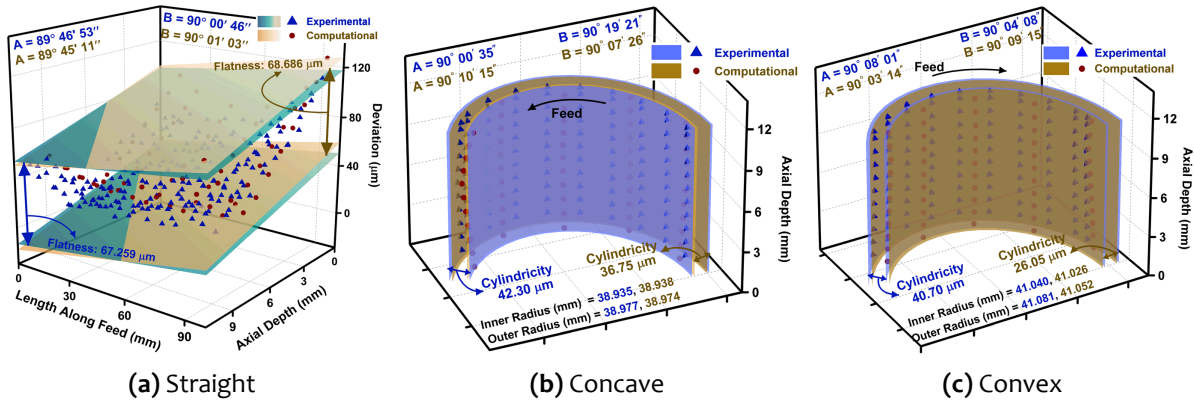


Figure 4.12 : Comparison of Geometric Tolerance Variation for Straight and Circular Geometries
Test 6 - *RDOC* = 4 mm; *ADOC* = 11 mm

The present thesis work also aim at realizing the relative contribution of tool and workpiece deflections to the geometric tolerances with change of *ADOC* (tests 5-8). The general understanding implies that the tool deflections will be significant at higher *ADOC* and contributes to the geometric tolerances. The results highlighted that the relative contribution of tool deflections is considerably lower at higher *ADOC*. The amount of work material being removed increases with *ADOC* resulting into significant reduction of the workpiece rigidity with the progress of machining leading to increased deflections of the component. The increased contribution of workpiece deflections can be attributed to considerable thinning of the component with the progress of material removal. The increased thinning of the component during the operation results into widening of the distorted point-cloud along length of the cut. The wider range of error variation is reflected in the form of elevated geometric tolerances at these cutting conditions. Based on outcomes of the present thesis work, it can be concluded that static deflections of both tool and workpiece contribute considerably to the geometric tolerances under different cutting conditions. Therefore, it is necessary to incorporate both deflection models in the computational framework for the improved prediction of the geometric tolerances.

4.4.2 Effect of Component Geometry

The curved geometries can be machined from either concave or convex side. It was reported in previous studies that the engagement area or cutter-workpiece transition zone alters significantly with the change of machining side with other cutting conditions at the same level. The concave side offers larger cutter-workpiece engagement area in comparison to straight or convex geometries under identical cutting conditions. The larger engagement area is an indicator of more material being removed for identical cutting conditions in the case of concave geometries implying higher cutting forces. The opposite behaviour is expected for convex geometries with lower cutting forces. The present subsection analyzes geometric tolerances for three different configurations of the thin-walled geometries; straight, circular concave and convex with the identical cutting conditions.

Figure 4.13 shows geometric tolerance and associated parameters obtained from computational models and machining experiments corresponding to Test 3 for the above three geometries under consideration. It can be seen that planar components have poor geometric tolerance in comparison to circular concave or convex geometries. Although concave components experience considerably larger engagement area, chip load and cutting force in comparison to straight geometry, the geometric accuracy of the components is much better. Such behaviour can be attributed to the structural configuration of concave structure which is inherently stiffer in comparison to straight counterparts that results into smaller deflections and thereby improved geometric tolerance. This is a favourable condition for the process planner as it permits use of more productive cutting conditions while machining concave geometries. Another important point to note here is the magnitude of geometric tolerance in concave and convex case which is comparable to each other even though the convex geometry experiences much lesser engagement area, chip load and cutting forces. Based on the general comprehension with reference to engagement area and cutting forces, it was expected that geometric tolerances for the concave geometry would be much higher than convex counterpart but the same is not observed. The effect of engagement area has been analyzed further in the subsequent subsection to investigate this aspect in a greater detail.

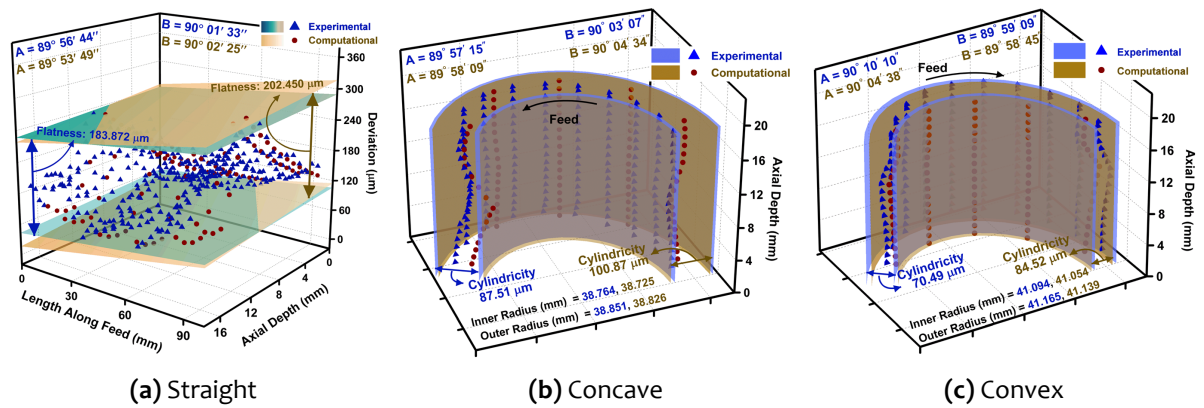


Figure 4.13 : Effect of Component Geometry on Geometric Tolerances
Test 3 - RDOC = 1.5mm; ADOC = 18 mm

4.4.3 Analysis of Cutter-Workpiece Interaction Area

The configurations of thin-walled components outlined in the previous section were considered for the analysis of cutter-workpiece engagement area under identical machining conditions. The engagement area was considerably different in each case due to the variation of workpiece geometry. In order to investigate this aspect further, the concept of “Equivalent RDOC” has been devised in this thesis work by extending the concept of constant engagement area [Desai and Rao, 2016]. The “Equivalent RDOC” represents the RDOC at which radial engagement angle is identical for both straight and circular geometries. Figure 4.14 depicts the schematic illustration to understand the concept of “Equivalent RDOC”. Figure 4.14 represents machining at variable RDOC for different geometric configurations where the engagement angle is maintained at the identical level in each case. The programmed RDOC in each case is different due to the nature of workpiece i.e. straight, concave or convex geometries. The RDOC value in each case is termed as “Equivalent RDOC” which offers an identical radial engagement for all three geometries. The concept of “Equivalent RDOC” is considered in this study by employing cutting conditions corresponding to Test 7 in Table 4.3. The “Equivalent RDOC” can be determined following two step approach,

1. The engagement angle is determined in case of straight geometries for the given RDOC and ADOC values using Eq. 4.20

$$\theta_{en} = \cos^{-1} \left(1 - \frac{RDOC}{R_c} \right) \quad (4.20)$$

2. The engagement angle determined for the straight geometry is maintained in case of circular concave and convex geometries by solving the following Eqs.4.21 and 4.22 for “Equivalent RDOC” (R_{eq}) value.

$$\theta_{en} = \cos^{-1} \left(\frac{R_c^2 + (R_f - R_c)^2 - (R_f - R_{eq})^2}{2R_c(R_f - R_c)} \right) \quad (4.21)$$

$$\theta_{en} = \cos^{-1} \left(\frac{R_c^2 + (R_f + R_c)^2 - (R_f + R_{eq})^2}{2R_c(R_f + R_c)} \right) \quad (4.22)$$

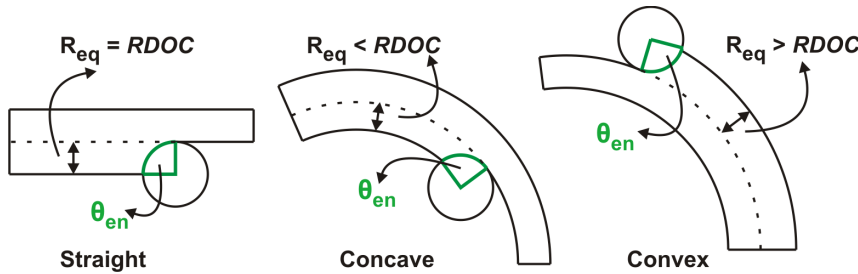


Figure 4.14 : Illustration of Constant Engagement for Geometries at “Equivalent RDOC”

The numerical values of “Equivalent RDOC” for different geometries are determined and listed in Table 4.6. The convex geometry has larger RDOC while concave geometry has smaller RDOC than the straight components to maintain the same engagement angle. These values are termed as “Equivalent RDOC” as it offers constant engagement angle for each case. It is essential to regulate the initial or unmachined thickness (D) of the component to achieve constant engagement angle and identical final thickness in each case. It implies reduction of the unmachined thickness (D) for concave geometry resulting into diminishing of workpiece rigidity. Similarly, the unmachined thickness (D) increases in case of convex geometry leading to enhanced rigidity for unmachined components. Such variation of initial workpiece rigidity due to different RDOC affects the deflection pattern and tolerances achievable for the machined components. Table 4.6 summarizes resultant dimensions of the unmachined component for different geometries corresponding to cutting conditions for Test 7.

Table 4.6 : Workpiece Dimensions and Cutting Condition for “Equivalent RDOC”

Geometry	Radius (mm)		Workpiece Thickness (D)	“Equivalent RDOC” (R_{eq})	Geometric Tolerance (μm)	
	Inner	Outer			Computational	Experimental
Straight	NA	NA	6 mm	4 mm	85.55	85.06
Concave	35.68	41	5.32 mm	3.32 mm	46.12	60.68
Convex	39	45.53	6.53 mm	4.53 mm	36.90	49.59
Engagement Angle (θ_{en}) : 60°						

Figure 4.15 shows the comparison of geometric tolerance obtained for straight, circular concave and convex geometries upon machining at “Equivalent RDOC” values. It can be seen that

the geometric tolerance is much higher for straight geometry in comparison to circular concave and convex geometries owing to lesser stiffness. Also, the convex geometry have lesser geometric tolerance in comparison to concave geometries. The smaller value of geometric tolerance for convex geometries can be attributed to enhanced rigidity of the component and reduced workpiece deflections. Similarly, concave geometries deflect more and results in to higher geometric tolerance due to lower rigidity of the component. It can also be inferred that the increase of cutting forces due to higher *RDOC* in case of convex geometries do not have significant effect on the workpiece deflections and geometric tolerance. Based on the discussions presented in this sections, it can be concluded that convex geometries present inherent machining advantage for process planners. The convex geometries can be machined at more productive elevated cutting conditions thereby offering better machining productivity.

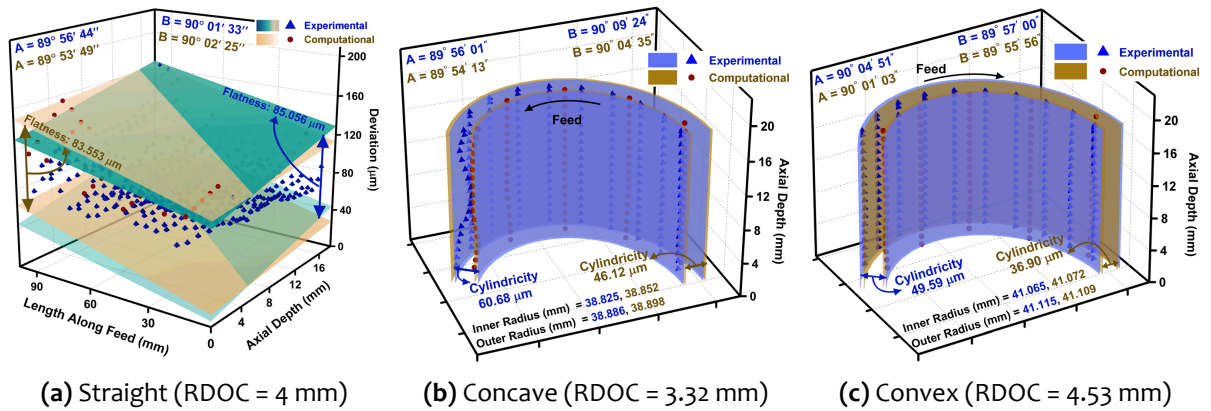


Figure 4.15 : Analysis of Constant Engagement Area on Geometric Tolerances
Test 7 - *ADOC* = 18 mm

4.4.4 Effect of Workpiece Curvature

The thesis also analyzed the effect of workpiece curvature on cylindricity parameters during machining of thin-walled components. To investigate this aspect, dimensions of the thin-walled component are designed such that the volume of material resisting cutting forces is maintained at constant level. It is achieved by maintaining dimensions of the component viz. thickness (*D*), height (*H*) and length (*L*) at the identical level with variation of workpiece radius. It results into thin-walled components with different workpiece curvature but constant volume of the material to resist machining forces. The study employs two different geometric configurations of circular concave and convex components with dimensions summarized in Table 4.7. The other cutting conditions such as *RDOC*, *ADOC*, workpiece geometry, cutting tool parameters, etc. are maintained at the constant level as summarized in Table 4.2 and 4.3 for cutting conditions corresponding to test 7.

Table 4.7 : Workpiece Dimensions and Cylindricity Values for Components with Varying Curvature

Geometry	Radius (mm)		Engagement Angle (θ_{en})	Geometric Tolerance (μm)	
	Inner	Outer		Computational	Experimental
Concave	55	61	63.82°	52.18	62.76
Convex	59	65	57.08°	48.02	56.96
Workpiece Dimension (L x H x D) (mm)			:	100 x 50 x 6	

Figure 4.16 shows the variation of geometric tolerance parameters for concave geometries at lower and higher (Test 1) workpiece curvature. It can be seen that the component with

lower workpiece curvature exhibits poor geometric accuracy of machined components. It can be attributed to the reduction of the component stiffness at lower workpiece curvature. The cutting force also reduces at lower workpiece curvature owing to the change of the engagement angle. The reduction of cutting force at lower workpiece curvature do not have underlying effect on the deflections in comparison to lower rigidity of the component. The change of workpiece curvature have marked effect on the rigidity and thereby geometric tolerance of the component. Figure 4.17 shows the comparison of geometric tolerance parameters for convex geometries with higher and lower workpiece curvature. It can be seen that the convex component with lower workpiece curvature exhibits poor geometric accuracy of machined components. It can be attributed to two factors; (i) the increased engagement area leading to increase in cutting forces on the component, and (ii) decrease in component stiffness with lower workpiece curvature. The change of workpiece curvature have marked effect on the rigidity and the magnitude of cutting force leading to higher geometric tolerance on the component.

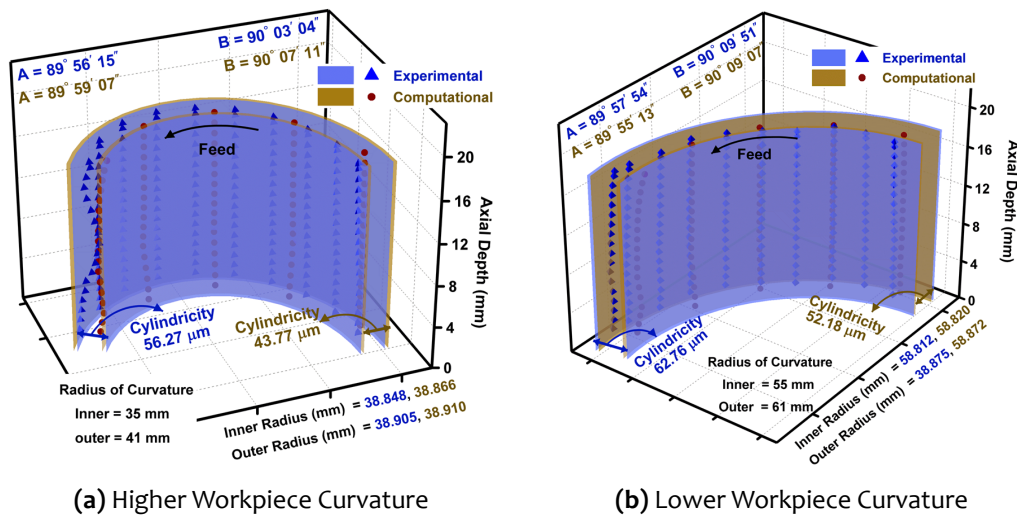


Figure 4.16 : Effect of Varying Workpiece Curvature for Concave Geometries
Test 7 - RDOC = 4 mm; ADOC = 18 mm

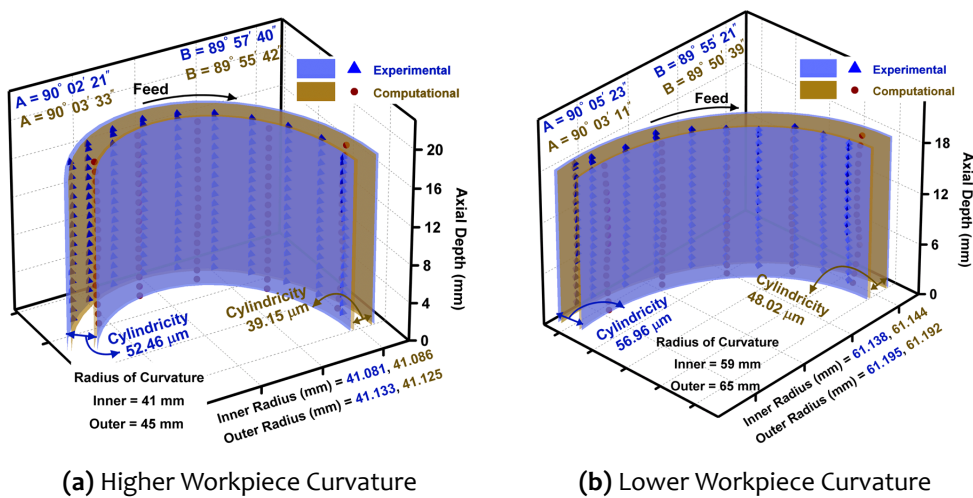


Figure 4.17 : Effect of Varying Workpiece Curvature for Convex Geometries
Test 7 - RDOC = 4 mm; ADOC = 18 mm

4.4.5 Effect of Workpiece Rigidity

The workpiece rigidity is another critical factor governing static deflections and thereby geometric tolerances for the thin-walled straight and circular components. The rigidity of the thin-walled component can be varied by altering the unmachined workpiece thickness. The increased unmachined thickness can provide adequate rigidity to thin-walled components and reduce static deflections. The present thesis work considers components having an identical final thickness and workpiece curvature but different rigidity under unmachined state to investigate this aspect further. The initial workpiece thickness is varied over a range to investigate the effect of component rigidity on the geometric tolerances. Table 4.6 outlines machining conditions (Test 5-8) for these studies where the unmachined thickness is varied from 3.5 to 6 mm, with the final thickness of the component maintained at 2 mm. The change of the unmachined thickness with constant final thickness increases *RDOC* from 1.5 to 4 mm. The increased *RDOC* results into elevated material removal rate and cutting forces during the machining operation.

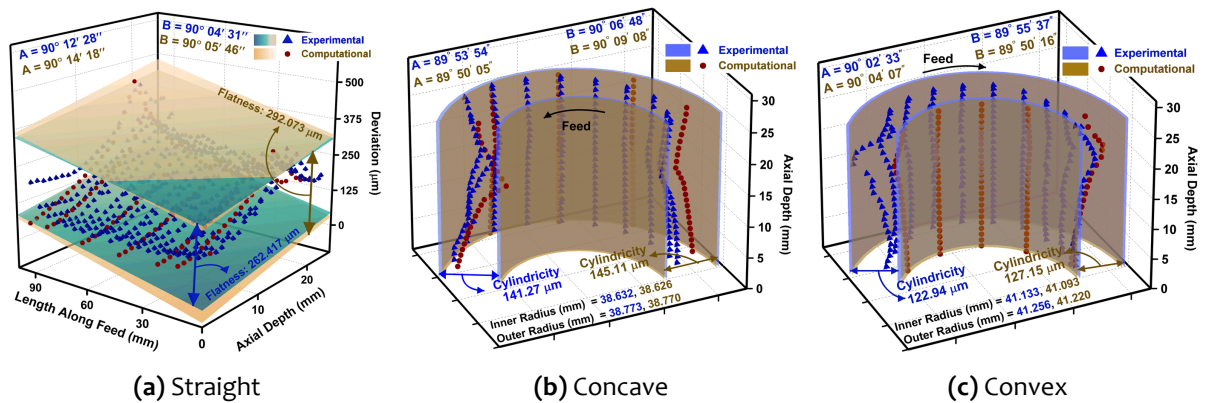


Figure 4.18 : Effect of Workpiece Rigidity on Geometric Tolerance
Test 4 - *RDOC* = 1.5 mm; *ADOC* = 25 mm

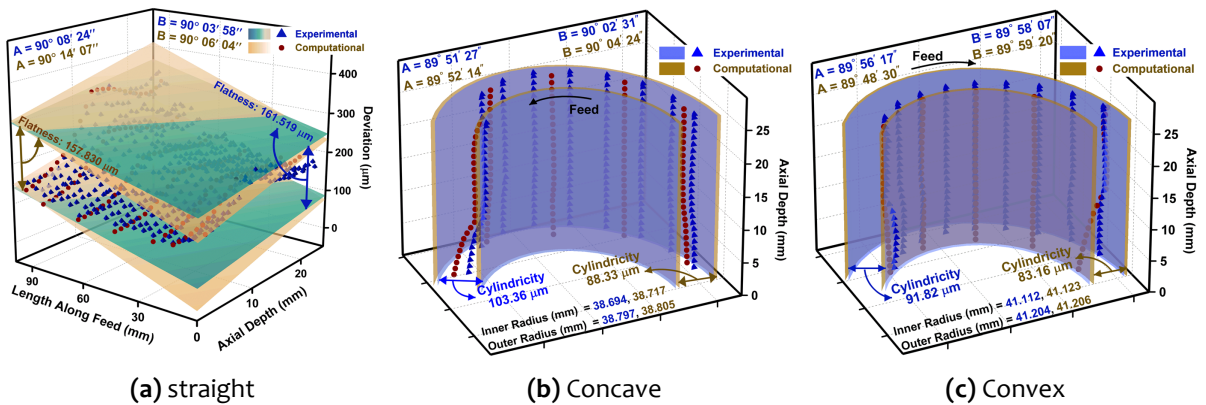


Figure 4.19 : Effect of Workpiece Rigidity on Geometric Tolerance
Test 8 - *RDOC* = 4 mm; *ADOC* = 25 mm

Figure 4.18 and 4.19 shows the effect of workpiece rigidity on geometric tolerances viz. flatness and cylindricity for conditions corresponding to Test 4 and 8. It is observed from computational and experimental results that the geometric tolerance is remarkably lower for cutting conditions (Test 8) involving higher material removal rates in comparison to conservative conditions (Test 4). The higher *RDOC* (Test 8) tends to increase cutting forces and tool deflections

but appreciably reduces workpiece deflections due to the unmachined component's higher rigidity. The cumulative effect of tool and workpiece deflections reduces total system deflections, thereby improving the geometric tolerances. It can be inferred that the rigidity of the component can be critical factor in determining geometric tolerances during the end milling of thin-walled components. A process planner can employ an appropriate balance of the productivity and static deflections in selecting optimum cutting conditions that achieve flatness and cylindricity values within limits specified by the designer.

4.5 SUMMARY

This chapter presented an overall framework to estimate geometric tolerances during the end milling of thin-walled straight and circular components. The computational framework combines computational elements such as mechanistic force model, FE-based workpiece deflection model, cantilever beam formulation based tool deflection model and PSO-based geometric tolerance estimation model. The framework incorporates important process linked issues viz. cutting force induced tool and workpiece deflections, nature of workpiece curvature (straight, concave and convex), component rigidity, etc. on geometric tolerances achievable during the operation. The proposed framework is implemented in the form of computational models and substantiated further by conducting end milling experiments on the thin-walled components over a wide range of cutting conditions. Thereafter, a comparison of geometric tolerances during end milling of straight and circular (convex or concave side machining) thin-walled component is presented. The chapter also investigates the effect of change in the radial engagement area due to workpiece curvature by introducing the concept of "*Equivalent RDOC*". Further, the effect of change in workpiece curvature is investigated on geometric tolerances in this chapter. Finally, the inevitable aspects of varying workpiece rigidity during end milling of thin-walled components on the geometric tolerances was also studied and it has been realized that geometric tolerances depends significantly on initial or unmachined state of the the component. The higher thickness of the unmachined workpiece increases rigidity considerably and reduces workpiece deflections. Therefore, unmachined state of component can be regulated effectively by process planners in controlling geometric tolerances during end milling of thin-walled components. The outcomes of the present study provide meaningful information to the process planners and assists in the selection of appropriate cutting conditions, tool-path planning, developing fixturing strategies/support systems, etc. during end milling of thin-walled components.

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