

Analyzing Physical Layer Security of PLC Systems Using DCSK: A Copula-Based Approach

This chapter ¹ analyzes the PLS performance of a DCSK modulation-based PLC system by exploiting the novel FGM Copula approach to model the dependence between the main and eavesdropper channels. A wiretap PLC channel model is investigated, where the main channel and the wiretap channel are assumed to be correlated and Log-normally distributed. The Gamma approximation to the Log-normal distribution is employed to simplify the computation. Concurrently, the PLC channel noise is modeled as a Bernoulli-Gaussian random process. Utilizing a Copula based approach to model the dependence among the correlated PLC channels, the PLS performance of the PLC system is evaluated in terms of SOP and SPSC. It is revealed through the asymptotic SOP analysis that the secrecy diversity order depends on the shaping parameter (m_{γ_M}) of the main channel. We also propose an algorithm to maximize the secrecy throughput under SOP constraints. Based on the insights from this analysis, it has been seen that the SOP performance degrades when the value of the dependence parameter (θ) increases. It is also revealed that the secrecy throughput performance improves with a lower optimal threshold value of the SNR, γ_{th} . Furthermore, some other insightful observations are presented by studying the impact of different parameters such as spreading factor, impulsive noise occurrence probability, transmitted power, and impulsive noise index.

2.1 Introduction

Over the past years, the fourth industrial revolution, also named as Industry 4.0, has amalgamated modern ICTs to design a complete, efficient, reliable, robust, optimized, and smart industrial system. Industry 4.0 mainly consists of cyber-physical systems and the IoT [Kalør *et al.*, 2018], where multiple devices, machines, vehicles, and buildings connect to each other via the integration of various ICTs. In Industry 4.0, ICTs are required to offer a very good quality-of-service (QoS) for the last mile access (“to the home”) and the last inch access (“in the home”) [Majumder and Caffery J, 2004]. QoS covers various key performance indicators (KPIs) such as better network connectivity, high data rates, low latency, high network coverage, low maintenance and installation cost, cybersecurity, etc. In this context, PLC is a suitable contender for fulfilling the above mentioned KPIs in Industry 4.0. PLC is a wireline technology that allows an exchange of digital data over the existing electric wires or power distribution networks. The notable advantage of PLC lies in its use of the existing ubiquitous wired infrastructure of the power grid, making it

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a low-cost technology and offering high network coverage. High data rates can be supported by PLC as demonstrated in HomePlug Powerline Alliance (HomePlug) and IEEE P1901 standards for home networking [Cano *et al.*, 2016]. Moreover, PLC can play a considerable role in IoT applications including smart buildings, smart vehicles, smart grids, and smart industries [Cano *et al.*, 2016]. Consequently, due to its appealing characteristics PLC technologies play a significant role in Industry 4.0 and IoT networks.

The security of data has always been a salient aspect of telecommunication systems. Serving an impenetrable security is considered an essential part of the QoS index. Specifically, in Industry 4.0 and IoT networks, various devices and machines interconnect through a broadcast communication network. Primarily, data security have been provided by cryptographic techniques and key management mechanisms. In IoT devices, various authentication schemes are being introduced. Recently, software-defined networking authentication schemes have been proposed to protect data in IoT devices [Salman *et al.*, 2016]. However, due to advancements in computer technology, these cryptographic techniques and key management mechanisms cannot provide unbreakable network security [Kartalopoulos, 2006a]. Following the early theoretical research conducted by Shannon, Wyner, Csiszár, and Körner [Csiszár and Körner, 1978]. In recent decades, there has been a renewed interest in the study of secrecy at the physical layer for both wireless and wireline channels to enhance the security of modern communication networks significantly [Mukherjee *et al.*, 2014]. In the PLS the randomness of the communication channels is exploited in the physical layer to prevent data leakage and it is contemplated as a complimentary and promising method that can strengthen digital data security.

The PLS performance of a wireless system and a SISO PLC system was examined and compared by the authors in [Pittolo and Tonello, 2013] and [Pittolo and Tonello, 2014]. It was evident that PLC systems had a lower secrecy rate than wireless communication. An evaluation of the PLS performance of a MIMO PLC systems was conducted in [Zhuang and Lampe, 2014]. It was determined in this study that MIMO PLC systems are more secure than SISO PLC systems. In [Salem *et al.*, 2017], the PLS performance of a cooperative PLC system was evaluated over correlated Log-normal communication channels.

Previously, numerous conventional modulation schemes have been incorporated into the design of PLC systems. Recently, PLC systems have adopted the DCSK modulation technique because of its benefits as mentioned in Chapter 1. The authors in [Kaddoum and Tadayon, 2017] examined the performance of DCSK-based PLC systems over the multipath power line channel model, considering background noise, impulsive noise, and phase noise. They found that DCSK schemes outperformed DS-DPSK and DS-CDMA schemes. In [Zheng *et al.*, 2017], the authors examined the effectiveness of M-ary DCSK in the presence of background, impulsive, and phase noise for the NB PLC systems. The study shows that M-DCSK performs better than DS-MDPSK. Additionally, the authors of [Zheng *et al.*, 2017] expanded their research in [Miao *et al.*, 2020] by designing and analyzing the replica piecewise (RP)-MDCSK system for PLS systems in the presence of asynchronous impulsive noise. Furthermore, the authors have shown that RP-MDCSK exhibits a much-reduced BER compared to DCSK and DS-MDPSK.

In recent times, Copula theory has received significant attention from researchers in communication channel modeling owing to its favorable characteristics, as discussed in Chapter 1. The authors of [Gholizadeh *et al.*, 2015] utilized the Copula approach to determine the capacity of a correlated ergodic MIMO channel over Nakagami- m fading. Three kinds of Copulas including, normal, Clayton, and t Copula were used to estimate the channel parameters. The authors utilized the survival Gumbel Copula approach to compute a closed-form expression for OP under cross-correlated Weibull fading with the selection combining technique [Huang *et al.*, 2015]. In [Mathur *et al.*, 2015], the authors calculated the average bit error rate (ABER) and ASER for PLC systems.

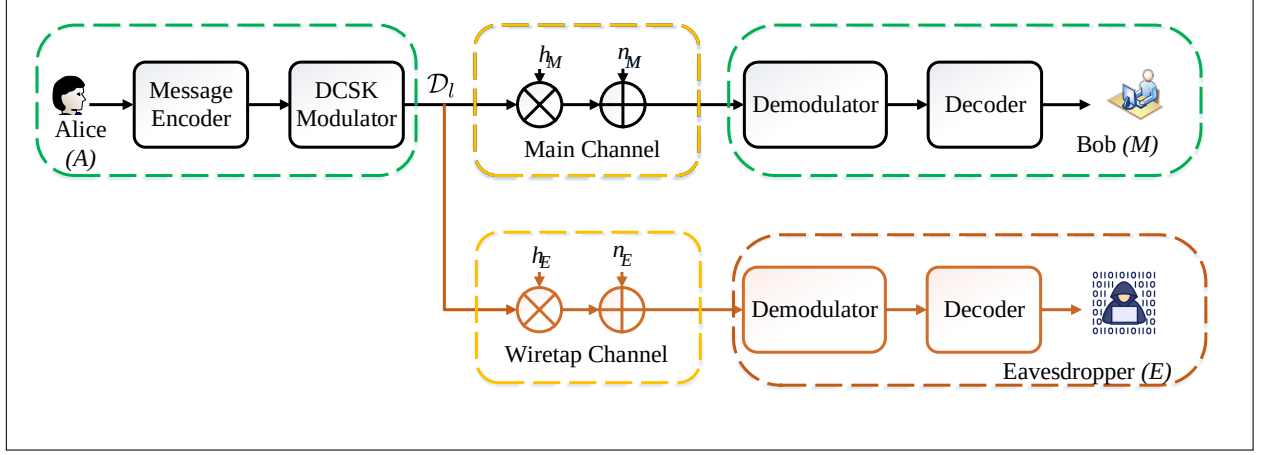


Figure 2.1: Wiretap power line channel system model.

They utilized a copula approach to analyze the dependence of decision variables and implemented a QPSK modulation scheme over Nakagami- m distributed additive background noise. Further, the authors in [Ghadi and Hodtani, 2020] established closed-form expressions for the coverage region and OP over correlated Rayleigh fading multiple access channels using the FGM Copula approach. Moreover, the PLS performance of a wireless communication system over correlated Rayleigh distributed wiretap fading channel was analyzed in [Ghadi and Hodtani, 2021], where closed-form expressions for the SCR, ASC, and SOP were derived utilizing the FGM Copula approach.

The rest of the chapter is arranged as follows: Section 2.2 presents the considered PLC system model, where the DCSK system, received power, PLC channel and noise models, and the characterization of the instantaneous SNR are described. After reviewing Copula theory, the detailed secrecy analysis of the proposed PLC system in terms of SOP and SPSC is presented in Section 2.3. Further, the asymptotic analysis of the SOP is conducted. We also carry out the secrecy throughput optimization analysis in Section 2.4. In Section 2.5, we demonstrate and discuss the numerical results while Section 2.6 presents a summary of this chapter.

2.2 System Model

Let us consider a discrete memoryless PLC wiretap channel model illustrated in Fig. 2.1, where Alice (A) transmits a message to an authorized receiver Bob (M) over the main channel. At the same time, a passive eavesdropper (E) wants to listen and attempts to decode the message of A via a wiretap power line channel.

2.2.1 Received Power

The received signals at M and E are given by

$$r_k = \delta_k \sqrt{P_{R_k}} h_k \mathcal{D}_l + n_k, k \in \{M, E\}. \quad (2.1)$$

In (2.1), $\delta_k = e^{-(a_0 + a_1 \cdot f^c)d}$ characterizes the attenuation of a power line link and a_0 , a_1 , and c are the attenuation parameters at a frequency f and path length d . h_M and h_E are the correlated complex channel gains of the main channel and wiretap channel, respectively, which are assumed to undergo flat fading. This is a valid assumption as the considered PLC network has a small number of branches with short lengths (0-300 m), and therefore the signal bandwidth is less than the channel's coherence bandwidth. Consequently, the PLC channels experience flat fading [Dubey *et al.*, 2014], [Mathur *et al.*, 2017]. In Eq. (2.1), n_M and n_E represent the channel noise on the main

channel and eavesdropper channel, respectively, and $\mathcal{D}_l$ denotes the single carrier DCSK modulated symbol transmitted by A .

2.2.2 PLC Channel Model

For the considered PLC system, we assume that both the main and wiretap channels are Log-normally distributed. The PDF of the PLC channel gain is expressed as [Mathur *et al.*, 2018, Eq. (4)]

$$f(h_k) = \frac{1}{h_k \sqrt{2\pi\sigma_k^2}} \exp\left(-\frac{(\ln h_k - \mu_k)^2}{2\sigma_k^2}\right), \quad h_k \geq 0, \quad k \in \{M, E\}. \quad (2.2)$$

In (2.2), μ_k and σ_k^2 are the mean and variance of the RV $\ln h_k$, respectively. Moreover, the CSI of the main and wiretap channels are assumed to be known. In our scenario, it is observed that carrying out the whole analysis with the Log-normal distribution in (2.2) is quite intractable to obtain the closed-form solution for the double integral expressions. Therefore, we utilize the Gamma approximation of the Log-normal distribution [Kostić, 2005, Eq. (3)]

$$f(h_k) \approx \frac{1}{\Gamma(m_{h_k})} \left(\frac{m_{h_k}}{\Omega_{h_k}}\right)^{m_{h_k}} h_k^{m_{h_k}-1} \exp\left(-\frac{m_{h_k} h_k}{\Omega_{h_k}}\right). \quad (2.3)$$

In (2.3), the parameters m_{h_k} and Ω_{h_k} hold the following relations with σ_k and μ_k :

$$m_{h_k} = \frac{1}{e^{\sigma_k^2} - 1} \quad \text{and} \quad \Omega_{h_k} = e^{\mu_k} \sqrt{\frac{m_{h_k} + 1}{m_{h_k}}}. \quad (2.4)$$

In (2.4), $k \in \{M, E\}$, m_{h_k} denotes the shaping parameter, and Ω_{h_k} is the mean power.

2.2.3 Noise Modeling

PLC channels generally yield a mixture of background noise and impulsive noise [Ferreira *et al.*, 2011]. The additive noise in a PLC system is usually modeled by the Bernoulli-Gaussian random process [Kaddoum and Tadayon, 2017]. Hence, the total noise can be described by

$$n_k = G'_{n_k} + I_{n_k}, \quad k \in \{M, E\}, \quad (2.5a)$$

$$I_{n_k} = P_B G''_{n_k}, \quad k \in \{M, E\}. \quad (2.5b)$$

In (2.5a) and (2.5b), G'_{n_k} and G''_{n_k} are Gaussian RVs corresponding to the background noise and impulsive noise component of the Bernoulli-Gaussian PLC noise having zero mean and variances σ_B^2 and σ_I^2 , respectively. I_{n_k} denotes the impulsive noise, where the Bernoulli RV (P_B) denotes the arrival of impulsive noise component of the PLC noise with the random amplitude of G''_{n_k} . Here, G'_{n_k} , G''_{n_k} , and P_B are assumed to be mutually independent. Thus, the PDF of the PLC channel noise on the main channel (A to M) and the eavesdropper channel (A to E) is given by [Mathur *et al.*, 2018, Eq. (5)]

$$f(n_k) = \sum_{j=1}^2 p_j \mathcal{N}(0, \sigma_{k_j}^2), \quad k \in \{M, E\}. \quad (2.6)$$

In (2.6), $p_1 = 1 - p$, $p_2 = p$, p is the probability of arrival of the impulsive noise, $\sigma_{k_1}^2 = \sigma_{B_k}^2$ is the background noise power, and $\sigma_{k_2}^2 = \sigma_{B_k}^2(1 + K)$ is the noise power of both background and impulsive noises. $K = \sigma_{I_k}^2 / \sigma_{B_k}^2$ denotes the the impulsive noise to the background noise power ratio.

2.2.4 Characterization of the Instantaneous SNR

The instantaneous SNR of the received DCSK signals at M and E can be written as [Mathur *et al.*, 2018, Eq. (6)]

$$\gamma_k = \begin{cases} \gamma_{k_1} = \frac{\delta_k^2 |h_k|^2 P_{R_k}}{2\beta \sigma_{B_k}^2} = \mathcal{H}_k \bar{\gamma}_k, & \text{only background noise,} \\ \gamma_{k_2} = \frac{\delta_k^2 |h_k|^2 P_{R_k}}{2\beta \sigma_{B_k}^2 (1+K)} = \mathcal{H}_k \bar{\gamma}'_k, & \text{with impulsive noise.} \end{cases} \quad (2.7)$$

In (2.7), $k \in \{M, E\}$, $\mathcal{H}_k = \delta_k^2 |h_k|^2 / (2\beta)$, $\bar{\gamma}_k = P_{R_k} / \sigma_{B_k}^2$, and $\bar{\gamma}'_k = \bar{\gamma}_k / (K+1)$. Further, the PDF of γ_{M_j} and γ_{E_j} can be written as

$$f_{\gamma_{k_j}}(\gamma_{k_j}) = \frac{1}{\Gamma(m_{\gamma_{k_j}})} \left(\frac{m_{\gamma_{k_j}}}{\Omega_{\gamma_{k_j}}} \right)^{m_{\gamma_{k_j}}} \gamma_{k_j}^{m_{\gamma_{k_j}}-1} \exp\left(-\frac{m_{\gamma_{k_j}} \gamma_{k_j}}{\Omega_{\gamma_{k_j}}}\right). \quad (2.8)$$

In (2.8), $k \in \{M, E\}$, $j \in \{1, 2\}$, $m_{\gamma_{k_j}} = 1/(e^{\sigma_{\gamma_{k_j}}^2} - 1)$, and $\Omega_{\gamma_{k_j}} = e^{\mu_{\gamma_{k_j}}} \sqrt{\frac{m_{\gamma_{k_j}} + 1}{m_{\gamma_{k_j}}}}$. Here, $\mu_{\gamma_{k_1}} = 2\mu_{h_k} + \ln(\delta_k^2 / (2\beta)) + \ln \bar{\gamma}_k$, $\mu_{\gamma_{k_2}} = 2\mu_{h_k} + \ln(\delta_k^2 / (2\beta)) + \ln \bar{\gamma}'_k$, and $\sigma_{\gamma_{M_j}}^2 = \sigma_{\gamma_{E_j}}^2 = 4\sigma_{h_k}^2$. From (2.8), the CDF of γ_{M_j} and γ_{E_j} can be expressed as

$$F_{\gamma_{k_j}}(\gamma_{k_j}) = \frac{1}{\Gamma(m_{\gamma_{k_j}})} \Gamma_l \left(m_{\gamma_{k_j}}, \frac{m_{\gamma_{k_j}} \gamma_{k_j}}{\Omega_{\gamma_{k_j}}} \right). \quad (2.9)$$

2.3 Physical Layer Security Analysis

In this section, we compute the PDF of SNR of the joint distribution of the main and eavesdropper channels. Further, capitalizing on that PDF, the PLS performance is measured by deriving series-based expressions of SOP, asymptotic SOP, and secrecy throughput optimization.

2.3.1 Computation of Joint PDF of γ_{M_j} and γ_{E_j}

According to the Sklar's theorem, the joint PDF of γ_{M_j} and γ_{E_j} can be written as

$$f_{\gamma_{M_j}, \gamma_{E_j}}(\gamma_{M_j}, \gamma_{E_j}) = f_{\gamma_{M_j}}(\gamma_{M_j}) f_{\gamma_{E_j}}(\gamma_{E_j}) \mathbb{C}_f(F_{\gamma_{M_j}}(\gamma_{M_j}), F_{\gamma_{E_j}}(\gamma_{E_j})). \quad (2.10)$$

Now, the FGM Copula function for γ_{M_j} and γ_{E_j} is defined as follows:

$$\mathbb{C}(F_{\gamma_{M_j}}(\gamma_{M_j}), F_{\gamma_{E_j}}(\gamma_{E_j})) = F_{\gamma_{M_j}}(\gamma_{M_j}) F_{\gamma_{E_j}}(\gamma_{E_j}) \left(1 + \theta (1 - F_{\gamma_{M_j}}(\gamma_{M_j})) (1 - F_{\gamma_{E_j}}(\gamma_{E_j})) \right). \quad (2.11)$$

In (2.11), $\theta \in [-1, 1]$ quantifies the dependency between the RVs γ_{M_j} and γ_{E_j} . A negative dependence between the two RVs corresponds to $\theta \in [-1, 0)$, $\theta \in (0, 1]$ represents positive dependence between the two RVs, while $\theta = 0$ denotes independence. Taking the partial derivative of (2.11), the FGM Copula density function can be calculated as

$$\begin{aligned} \mathbb{C}_f(F_{\gamma_{M_j}}(\gamma_{M_j}), F_{\gamma_{E_j}}(\gamma_{E_j})) &= \frac{\partial^2 \mathbb{C}(F_{\gamma_{M_j}}(\gamma_{M_j}), F_{\gamma_{E_j}}(\gamma_{E_j}))}{\partial \gamma_{M_j} \partial \gamma_{E_j}} \\ &= 1 + \theta \left((1 - 2F_{\gamma_{M_j}}(\gamma_{M_j})) (1 - 2F_{\gamma_{E_j}}(\gamma_{E_j})) \right). \end{aligned} \quad (2.12)$$

Finally, by substituting (2.12) into (2.10) and after some mathematical simplifications, the joint PDF of γ_{M_j} and γ_{E_j} can be expressed as

$$\begin{aligned} f_{\gamma_{M_j}, \gamma_{E_j}}(\gamma_{M_j}, \gamma_{E_j}) = & (1 + \theta) f_{\gamma_{M_j}}(\gamma_{M_j}) f_{\gamma_{E_j}}(\gamma_{E_j}) - 2\theta f_{\gamma_{M_j}}(\gamma_{M_j}) f_{\gamma_{E_j}}(\gamma_{E_j}) F_{\gamma_{M_j}}(\gamma_{M_j}) \\ & - 2\theta f_{\gamma_{M_j}}(\gamma_{M_j}) f_{\gamma_{E_j}}(\gamma_{E_j}) F_{\gamma_{E_j}}(\gamma_{E_j}) + 4\theta f_{\gamma_{M_j}}(\gamma_{M_j}) f_{\gamma_{E_j}}(\gamma_{E_j}) F_{\gamma_{M_j}}(\gamma_{M_j}) F_{\gamma_{E_j}}(\gamma_{E_j}). \end{aligned} \quad (2.13)$$

2.3.2 SOP Analysis

The *SOP* is the probability that the instantaneous secrecy capacity falls below a predefined threshold rate ($\mathcal{R}_{th}$) [Pan *et al.*, 2016]. By following the same assumption that was mentioned in the work of [Herath *et al.*, 2012; Sheikh-Hosseini *et al.*, 2016; Vu *et al.*, 2014; Wiklundh *et al.*, 2009; Dubey and Mallik, 2016; Salem *et al.*, 2017], we have assumed the upper bound on the channel capacity with full knowledge of the noise state available at the transmitter and receivers. Thus, the *SOP* of the considered system can be expressed as [Salem *et al.*, 2017], [Sheikh-Hosseini *et al.*, 2016], [Dubey and Mallik, 2016], [Mohan *et al.*, 2019]

$$SOP = \sum_{j=1}^2 p_j \times P_{sop_j}. \quad (2.14)$$

In (2.14),

$$\begin{aligned} P_{sop_j} &= [\Pr \{ \mathcal{C}_s(\gamma_{M_j}, \gamma_{E_j}) < \mathcal{R}_{th} \}] = \left[\Pr \left(\frac{1 + \gamma_{M_j}}{1 + \gamma_{E_j}} < \exp(\mathcal{R}_{th}) \right) \right] \\ &= [\Pr(\gamma_{M_j} < \lambda \gamma_{E_j} + \lambda - 1)]. \end{aligned} \quad (2.15)$$

In (2.15), $\mathcal{C}_s$ represents the instantaneous secrecy capacity and $\lambda = \exp(\mathcal{R}_{th})$. Further, the exact *SOP* analysis is intractable; due to this, obtaining the closed-form solution of the double integral expressions following (2.15) becomes quite difficult. Therefore, we proceed with computing the lower bound on *SOP*. In this context, assuming $\lambda(\gamma_{E_j} + 1) - 1 \approx \lambda \gamma_{E_j}$, we get

$$\begin{aligned} P_{sop_j} &\approx [\Pr \{ \gamma_{M_j} < \lambda \gamma_{E_j} \}] \\ &\approx \int_0^\infty \int_0^{\lambda \gamma_{E_j}} f_{\gamma_{M_j}, \gamma_{E_j}}(\gamma_{M_j}, \gamma_{E_j}) d\gamma_{M_j} d\gamma_{E_j}. \end{aligned} \quad (2.16)$$

Substituting (2.13) into (2.16), we have

$$P_{sop_j} \approx P_{1,j} - P_{2,j} - P_{3,j} + P_{4,j}. \quad (2.17)$$

In (2.17),

$$P_{1,j} = (1 + \theta) \int_0^\infty \int_0^{\lambda \gamma_{E_j}} f_{M_j}(\gamma_{M_j}) f_{E_j}(\gamma_{E_j}) d\gamma_{M_j} d\gamma_{E_j}, \quad (2.18)$$

$$P_{2,j} = 2\theta \int_0^\infty \int_0^{\lambda \gamma_{E_j}} f_{M_j}(\gamma_{M_j}) f_{E_j}(\gamma_{E_j}) F_{M_j}(\gamma_{M_j}) d\gamma_{M_j} d\gamma_{E_j}, \quad (2.19)$$

$$P_{3,j} = 2\theta \int_0^\infty \int_0^{\lambda \gamma_{E_j}} f_{M_j}(\gamma_{M_j}) f_{E_j}(\gamma_{E_j}) F_{E_j}(\gamma_{E_j}) d\gamma_{M_j} d\gamma_{E_j}, \quad (2.20)$$

$$P_{4,j} = 4\theta \int_0^\infty \int_0^{\lambda \gamma_{E_j}} f_{M_j}(\gamma_{M_j}) f_{E_j}(\gamma_{E_j}) F_{M_j}(\gamma_{M_j}) F_{E_j}(\gamma_{E_j}) d\gamma_{M_j} d\gamma_{E_j}. \quad (2.21)$$

In the following **Theorems 2.1-2.4**, we derive the series-based expressions for (2.18)-(2.21).

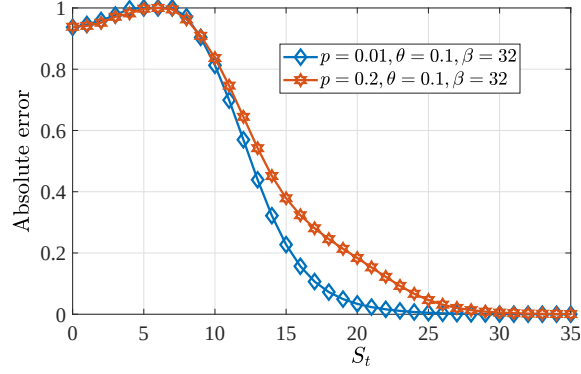


Figure 2.2: Absolute error versus S_t .

Theorem 2.1. *The series-based expression of $P_{1,j}$ is given as*

$$P_{1,j} = \frac{1 + \theta}{\Gamma(m_{\gamma_{M_j}})\Gamma(m_{\gamma_{E_j}})} \chi^{m_{\gamma_{E_j}}} \left(\frac{\Gamma(\xi)}{m_{\gamma_{M_j}} \eta^\xi} \right) \zeta^{m_{\gamma_{M_j}}} {}_2F_1 \left(1, \xi; m_{\gamma_{M_j}} + 1; \frac{\zeta}{\eta} \right), \quad (2.22)$$

where $\chi = \left(\frac{m_{\gamma_{E_j}}}{\Omega_{\gamma_{E_j}}} \right)$, $\zeta = \left(\frac{m_{\gamma_{M_j}} \lambda}{\Omega_{\gamma_{M_j}}} \right)$, $\xi = (m_{\gamma_{M_j}} + m_{\gamma_{E_j}})$, and $\eta = \left(\frac{m_{\gamma_{E_j}}}{\Omega_{\gamma_{E_j}}} + \frac{m_{\gamma_{M_j}} \lambda}{\Omega_{\gamma_{M_j}}} \right)$.

Proof. Refer to Appendix for the proof. ■

Theorem 2.2. *The series-based expression of $P_{2,j}$ is expressed as*

$$P_{2,j} = \frac{(2\theta)\chi^{m_{\gamma_{E_j}}}}{[\Gamma(m_{\gamma_{M_j}})]^2\Gamma(m_{\gamma_{E_j}})} \sum_{\ell=0}^{\infty} \left[\frac{(-1)^\ell}{\ell(m_{\gamma_{E_j}} + \ell)} \zeta^{\tilde{a}} \left(\frac{\Gamma(\tilde{b})}{\tilde{a}\eta^{\tilde{b}}} \right) {}_2F_1 \left(1, \tilde{b}; \tilde{a} + 1; \frac{\zeta}{\eta} \right) \right], \quad (2.23)$$

where $\tilde{b} = (2m_{\gamma_{M_j}} + m_{\gamma_{E_j}} + \ell)$ and $\tilde{a} = (2m_{\gamma_{M_j}} + \ell)$.

Proof. Refer to Appendix for the proof. ■

Theorem 2.3. *The series-based expression of $P_{3,j}$ is derived as*

$$P_{3,j} = \frac{(2\theta)\zeta^{m_{\gamma_{M_j}}}}{\Gamma(m_{\gamma_{M_j}})[\Gamma(m_{\gamma_{E_j}})]^2} \sum_{\ell=0}^{\infty} \left[\frac{(-1)^\ell}{\ell(m_{\gamma_{E_j}} + \ell)} \chi^{(2m_{\gamma_{E_j}} + \ell)} \left(\frac{\Gamma(\tilde{d})}{m_{\gamma_{M_j}} \eta^{\tilde{d}}} \right) {}_2F_1 \left(1, \tilde{d}; m_{\gamma_{M_j}} + 1; \frac{\zeta}{\eta} \right) \right], \quad (2.24)$$

where $\tilde{d} = (m_{\gamma_{M_j}} + 2m_{\gamma_{E_j}} + \ell)$.

Proof. Refer to Appendix for the proof. ■

Theorem 2.4. *The series-based expression of $P_{4,j}$ is given as*

$$P_{4,j} = \frac{4\theta}{[\Gamma(m_{\gamma_{M_j}})]^2[\Gamma(m_{\gamma_{E_j}})]^2} \sum_{\ell=0}^{\infty} \sum_{\psi=0}^{\infty} \left[\frac{(-1)^\ell}{\ell! \tilde{r}} \frac{(-1)^\psi}{\psi! \tilde{s}} \chi^{\tilde{t}} \left(\frac{\Gamma(\kappa) \zeta^{\tilde{a}}}{\tilde{a} \eta^\kappa} \right) {}_2F_1 \left(1, \kappa; \tilde{a} + 1; \frac{\zeta}{\eta} \right) \right], \quad (2.25)$$

where $\tilde{r} = (m_{\gamma_{M_j}} + \ell)$, $\tilde{s} = (m_{\gamma_{E_j}} + \psi)$, $\tilde{t} = (2m_{\gamma_{E_j}} + \psi)$, and $\kappa = (2m_{\gamma_{M_j}} + 2m_{\gamma_{E_j}} + \ell + \psi)$.

Proof. Refer to Appendix for the proof. ■

2.3.3 Convergence Analysis

On substituting (2.22)-(2.25) into (2.14), the series-based expressions of the *SOP* lower bound is obtained. It is observed from (2.23)-(2.25) that the series-based expressions for $P_{2,j}$, $P_{3,j}$, and $P_{4,j}$ consist of infinite summations. Let us present the convergence analysis of the infinite series here. In order to verify the convergence of (2.14), we calculate the absolute error, defined as

$$\text{Absolute error} = |\text{Exact value} - \text{Approximated value}|. \quad (2.26)$$

In (2.26), the exact value is obtained from (2.14) and the approximated value is computed from (2.17) using (2.22)-(2.25). Fig. 2.2 shows the plot of the absolute error against the number of summation terms (S_t) for various values of p , when $\beta = 32, \theta = 0.1, K = 100$, and the transmitted power on the A to M and A to E links is fixed at 20 dB and 10 dB, respectively. It can be noticed from the figure that as the value of S_t increases, the absolute error tends to zero. This demonstrates the convergence of the summation terms in the infinite series of (2.14). Therefore, in what follows, we consider $S_t = 33$ for our analysis.

2.3.4 Asymptotic SOP Analysis

To gain more insights into the secrecy performance analysis, we analyze the asymptotic behavior of the *SOP* in the high SNR regime. To this end, we start by examining (2.17) after substituting the series-based expressions of $P_{i,j}$, $i = \{1, 2, 3, 4\}$ from (2.22)-(2.25) for high average SNR on the main channel. As $\bar{\gamma}_M \rightarrow \infty$, the generalized Hypergeometric function in (2.22)-(2.25) tends to unity, i.e., ${}_2F_1(\cdot, \cdot; \cdot; \cdot) \rightarrow 1$. The infinite summation vanishes in (2.23)-(2.25) because, at high average SNR, only the terms corresponding to $\ell = \psi = 0$ dominate. Hence, the terms $P_{1,j}, P_{2,j}, P_{3,j}$, and $P_{4,j}$, can be approximated as shown in (2.27)-(2.30):

$$P_{1,j} \approx \mathcal{C}_{g1,j} \times \left(\frac{1}{\Omega_{\gamma_{M_j}}} \right)^{\overbrace{m_{\gamma_{M_j}}}^{D_{o1,j}}}, \quad (2.27)$$

$$P_{2,j} \approx \mathcal{C}_{g2,j} \times \left(\frac{1}{\Omega_{\gamma_{M_j}}} \right)^{\overbrace{2m_{\gamma_{M_j}}}^{D_{o2,j}}}, \quad (2.28)$$

$$P_{3,j} \approx \mathcal{C}_{g3,j} \times \left(\frac{1}{\Omega_{\gamma_{M_j}}} \right)^{\overbrace{m_{\gamma_{M_j}}}^{D_{o3,j}}}, \quad (2.29)$$

$$P_{4,j} \approx \mathcal{C}_{g4,j} \times \left(\frac{1}{\Omega_{\gamma_{M_j}}} \right)^{\overbrace{2m_{\gamma_{M_j}}}^{D_{o4,j}}}. \quad (2.30)$$

In (2.27)-(2.30),

$$\mathcal{C}_{g1,j} = \frac{(1 + \theta)\chi^{m_{\gamma_{E_j}}}}{\Gamma(m_{\gamma_{M_j}})\Gamma(m_{\gamma_{E_j}})} \left(\frac{(m_{\gamma_{M_j}}\lambda)^{m_{\gamma_{M_j}}}\Gamma(\xi)}{m_{\gamma_{M_j}}\chi^\xi} \right), \quad (2.31a)$$

$$\mathcal{C}_{g2,j} = \frac{(2\theta)\chi^{m_{\gamma_{E_j}}}}{[\Gamma(m_{\gamma_{M_j}})]^2\Gamma(m_{\gamma_{E_j}})} \left(\frac{(m_{\gamma_{M_j}}\lambda)^{2m_{\gamma_{M_j}}}\Gamma(b')}{(m_{\gamma_{E_j}})(2m_{\gamma_{M_j}})\chi^{b'}} \right), \quad (2.31b)$$

$$\mathcal{C}_{g_{3,j}} = \frac{(2\theta)\chi^{(2m_{\gamma_{E_j}})}}{\Gamma(m_{\gamma_{M_j}})[\Gamma(m_{\gamma_{E_j}})]^2} \left(\frac{(m_{\gamma_{M_j}}\lambda)^{m_{\gamma_{M_j}}}\Gamma(d')}{(m_{\gamma_{E_j}})(m_{\gamma_{M_j}})\chi^{d'}} \right), \quad (2.31c)$$

$$\mathcal{C}_{g_{4,j}} = \frac{(2\theta)\chi^{(2m_{\gamma_{E_j}})}}{[\Gamma(m_{\gamma_{M_j}})]^2[\Gamma(m_{\gamma_{E_j}})]^2} \left(\frac{(m_{\gamma_{M_j}}\lambda)^{2m_{\gamma_{M_j}}}\Gamma(\kappa')}{(m_{\gamma_{E_j}})(2m_{\gamma_{M_j}})^2\chi^{\kappa'}} \right). \quad (2.31d)$$

In (2.31b), (2.31c), and (2.31d), $b' = (2m_{\gamma_{M_j}} + m_{\gamma_{E_j}})$, $d' = (m_{\gamma_{M_j}} + 2m_{\gamma_{E_j}})$, and $\kappa' = (2m_{\gamma_{M_j}} + 2m_{\gamma_{E_j}})$, respectively. Finally, substituting (2.27)-(2.30) into (2.14), the series-based expressions of the asymptotic *SOP* can be computed.

It is known that the asymptotic analysis of any system is characterized by two parameters: coding gain ($\mathcal{G}_c$) and diversity order ($\mathcal{D}_o$). The coding gain shows the relative horizontal shift of the *SOP* versus SNR plot on a log-log scale. The diversity order specifies the asymptotic slope of the curve at high SNR values. The relation between coding gain and diversity order for the considered PLC system is computed from (2.27)-(2.30) as

$$\begin{aligned} \lim_{\bar{\gamma}_M \rightarrow \infty} SOP \approx \sum_{j=1}^2 p_j \left[\mathcal{G}_{c_{1,j}} \left(\frac{1}{\Omega_{\gamma_{M_j}}(\bar{\gamma}_M)} \right)^{\mathcal{D}_{o_{1,j}}} - \mathcal{G}_{c_{2,j}} \left(\frac{1}{\Omega_{\gamma_{M_j}}(\bar{\gamma}_M)} \right)^{\mathcal{D}_{o_{2,j}}} \right. \\ \left. - \mathcal{G}_{c_{3,j}} \left(\frac{1}{\Omega_{\gamma_{M_j}}(\bar{\gamma}_M)} \right)^{\mathcal{D}_{o_{3,j}}} + \mathcal{G}_{c_{4,j}} \left(\frac{1}{\Omega_{\gamma_{M_j}}(\bar{\gamma}_M)} \right)^{\mathcal{D}_{o_{4,j}}} \right]. \end{aligned} \quad (2.32)$$

Furthermore, from (2.32), the overall diversity order is obtained as $\mathcal{D}_o = \min\{\mathcal{D}_{o_{1,j}}, \mathcal{D}_{o_{2,j}}, \mathcal{D}_{o_{3,j}}, \mathcal{D}_{o_{4,j}}\} = \min\{\mathcal{D}_{o_{1,j}}, \mathcal{D}_{o_{3,j}}\}$. Thus, (2.32) can be re-written as

$$\lim_{\bar{\gamma}_M \rightarrow \infty} SOP \approx \sum_{j=1}^2 p_j \left[\underbrace{(\mathcal{G}_{c_{1,j}} - \mathcal{G}_{c_{3,j}})}_{\mathcal{G}_{c,j}} \left(\frac{1}{\Omega_{\gamma_{M_j}}(\bar{\gamma}_M)} \right)^{\mathcal{D}_{o_{1,j}}} \right]. \quad (2.33)$$

In (2.33), $\mathcal{D}_{o_{1,j}} = \mathcal{D}_{o_{3,j}} = m_{\gamma_{M_j}}$.

2.3.5 SPSC Analysis

In order to guarantee perfect secrecy over the communication channel, the secrecy capacity must be a non-zero quantity, which is also interpreted as *SPSC*. The *SPSC* for the considered PLC system is given by

$$SPSC = \sum_{j=1}^2 p_j \times P_{spsc_j}. \quad (2.34)$$

In (2.34),

$$\begin{aligned} P_{spsc_j} &= \Pr(\gamma_{M_j} > \gamma_{E_j}) = 1 - \Pr(\gamma_{M_j} \leq \gamma_{E_j}) \\ &= 1 - \int_0^\infty \int_0^{\gamma_{E_j}} f_{\gamma_{M_j}, \gamma_{E_j}}(\gamma_{M_j}, \gamma_{E_j}) d\gamma_{M_j} d\gamma_{E_j}. \end{aligned} \quad (2.35)$$

Here, the integral in (2.35) can be solved using (2.13) as

$$P_{spsc_j} = 1 - (P'_{1,j} - P'_{2,j} - P'_{3,j} + P'_{4,j}). \quad (2.36)$$

In (2.36), the series-based expressions for $P'_{1,j}$, $P'_{2,j}$, $P'_{3,j}$, and $P'_{4,j}$ can be calculated similar to (2.22)-(2.25) by substituting $\lambda = 0$ in (2.22)-(2.25) as follows:

$$P'_{1,j} = \frac{1 + \theta}{\Gamma(m_{\gamma_{M_j}})\Gamma(m_{\gamma_{E_j}})} \chi^{m_{\gamma_{E_j}}} \left(\frac{\Gamma(\xi)}{m_{\gamma_{M_j}}(\eta')^\xi} \right) (\zeta')^{m_{\gamma_{M_j}}} {}_2F_1 \left(1, \xi; m_{\gamma_{M_j}} + 1; \frac{\zeta'}{\eta'} \right), \quad (2.37)$$

$$P'_{2,j} = \frac{(2\theta)\chi^{m_{\gamma_{E_j}}}}{[\Gamma(m_{\gamma_{M_j}})]^2\Gamma(m_{\gamma_{E_j}})} \sum_{\ell=0}^{\infty} \left[\frac{(-1)^\ell}{\ell!(m_{\gamma_{E_j}} + \ell)} (\zeta')^{\tilde{a}} \left(\frac{\Gamma(\tilde{b})}{\tilde{b}(\eta')^{\tilde{b}}} \right) {}_2F_1 \left(1, \tilde{b}; \tilde{a} + 1; \frac{\zeta'}{\eta'} \right) \right], \quad (2.38)$$

$$P'_{3,j} = \frac{(2\theta)(\zeta')^{m_{\gamma_{M_j}}}}{\Gamma(m_{\gamma_{M_j}})[\Gamma(m_{\gamma_{E_j}})]^2} \sum_{\ell=0}^{\infty} \left[\frac{(-1)^\ell}{\ell!(m_{\gamma_{E_j}} + \ell)} \chi^{(2m_{\gamma_{E_j}} + \ell)} \left(\frac{\Gamma(\tilde{d})}{m_{\gamma_{M_j}}(\eta')^{\tilde{d}}} \right) {}_2F_1 \left(1, \tilde{d}; m_{\gamma_{M_j}} + 1; \frac{\zeta'}{\eta'} \right) \right], \quad (2.39)$$

$$P'_{4,j} = \frac{4\theta}{\Gamma(m_{\gamma_{M_j}})^2\Gamma(m_{\gamma_{E_j}})^2} \sum_{\ell=0}^{\infty} \sum_{\psi=0}^{\infty} \left[\frac{(-1)^\ell}{\ell!\tilde{r}} \frac{(-1)^\psi}{\psi!\tilde{s}} \chi^{\tilde{t}} \left(\frac{\Gamma(\kappa)(\zeta')^{\tilde{a}}}{\tilde{a}(\eta')^\kappa} \right) {}_2F_1 \left(1, \kappa; \tilde{a} + 1; \frac{\zeta'}{\eta'} \right) \right]. \quad (2.40)$$

In (2.40), $\zeta' = \left(\frac{m_{\gamma_{M_j}}}{\Omega_{\gamma_{M_j}}} \right)$ and $\eta' = \left(\frac{m_{\gamma_{E_j}}}{\Omega_{\gamma_{E_j}}} + \frac{m_{\gamma_{M_j}}}{\Omega_{\gamma_{M_j}}} \right)$.

2.4 Secrecy Throughput Optimization Analysis

In this section, we consider the secrecy throughput optimization for the considered PLC system. Our objective is to maximize the secrecy throughput under the *SOP* constraints; an algorithm is proposed for the same in this subsection.

Let us assume the Wyner's wiretap encoding scheme for the considered system model, where A transmits information at rate R_M to M by maintaining a predefined secrecy rate $\mathcal{R}_{th}$ against E . The net rate difference $R_D = R_M - \mathcal{R}_{th}$ estimates the cost of securing data transmission. For perfect secrecy, the main channel (A to M link) must follow the condition $C_M > \mathcal{R}_{th}$. Otherwise, perfect secrecy fails, i.e. $C_E > R_D$, where C_M and C_E are the instantaneous channel capacities of M and E , respectively. Thus, the conditional *SOP* is formulated as [Zhou *et al.*, 2011]

$$P_{csop_j} \triangleq \Pr(C_{E_j} > R_{M_j} - \mathcal{R}_{th} | \text{data transmission}). \quad (2.41)$$

Using (2.41), the overall conditional *SOP* of the considered system can be written as

$$CSOP = \sum_{j=1}^2 p_j \times P_{csop_j}. \quad (2.42)$$

We aim to maximize the secrecy throughput for the considered PLC system. The secrecy throughput is defined as

$$\tau = \mathcal{P}_{tx} \mathcal{R}_{th}. \quad (2.43)$$

In (2.43), $\mathcal{P}_{tx}$ denotes the transmission probability. Hence, the optimization problem can be formulated as

$$\max \tau, \quad \text{s.t. } CSOP \leq \varrho, \mathcal{P}_{tx} \geq \nu. \quad (2.44)$$

In (2.44), $\varrho \in [0, 1]$ and $\nu \in [0, 1]$ represent the constraints for secure and reliable transmission, respectively. In addition, we assume that the data transmission is possible whenever the value of γ_M is greater than a certain threshold SNR, γ_{th} , with $\gamma_{\text{th}} \geq \exp(\mathcal{R}_{\text{th}}) - 1$. Therefore, the transmission probability can be expressed as

$$\begin{aligned} \mathcal{P}_{\text{tx}} &= \Pr(\gamma_{M_j} > \gamma_{\text{th}}) = \frac{1}{\Gamma(m_{\gamma_{M_j}})} \Gamma_u \left(m_{\gamma_{M_j}}, \frac{m_{\gamma_{M_j}} \gamma_{\text{th}}}{\Omega_{\gamma_{M_j}}} \right) \\ &= \frac{1}{\Gamma(m_{\gamma_{M_j}})} \left[\Gamma(m_{\gamma_{M_j}}) - \sum_{i=0}^{\infty} \frac{(-1)^i}{i!(m_{\gamma_{M_j}} + i)} \left(\frac{m_{\gamma_{M_j}} \gamma_{\text{th}}}{\Omega_{\gamma_{M_j}}} \right)^{m_{\gamma_{M_j}} + i} \right]. \end{aligned} \quad (2.45)$$

Let us evaluate the conditional *SOP* as follows:

$$\begin{aligned} P_{\text{csop}_j} &= \Pr(\mathbf{C}_{E_j} > \mathbf{C}_{M_j} | \gamma_{M_j} > \gamma_{\text{th}}) \\ &= \Pr(\gamma_{M_j} < \exp(\mathcal{R}_{\text{th}})(1 + \gamma_{E_j}) - 1 | \gamma_{M_j} > \gamma_{\text{th}}) \\ &= \frac{\Pr(\gamma_{\text{th}} < \gamma_{M_j} < \lambda(1 + \gamma_{E_j}) - 1)}{\Pr(\gamma_{M_j} > \gamma_{\text{th}})}. \end{aligned} \quad (2.46)$$

Considering the lower bound in (2.46), i.e., $\lambda(\gamma_{E_j} + 1) - 1 \approx \lambda\gamma_{E_j}$, we obtain

$$\begin{aligned} P_{\text{csop}_j} &\approx \frac{\Pr(\gamma_{\text{th}} < \gamma_{M_j} < \lambda\gamma_{E_j})}{\Pr(\gamma_{M_j} > \gamma_{\text{th}})} \\ &\approx \frac{\int_{\Theta}^{\infty} \int_{\gamma_{\text{th}}}^{\lambda\gamma_{E_j}} f_{\gamma_{M_j}, \gamma_{E_j}}(\gamma_{M_j}, \gamma_{E_j}) d\gamma_{M_j} d\gamma_{E_j}}{\Pr(\gamma_{M_j} > \gamma_{\text{th}})}. \end{aligned} \quad (2.47)$$

In (2.47), $\Theta = \left(\frac{1 + \gamma_{\text{th}}}{\lambda} \right) - 1$. Here, the double integral in (2.47) can be divided into two parts as follows:

$$\mathbb{I} = \underbrace{\int_{\Theta}^{\infty} \int_0^{\lambda\gamma_{E_j}} f_{\gamma_{M_j}, \gamma_{E_j}}(\gamma_{M_j}, \gamma_{E_j}) d\gamma_{M_j} d\gamma_{E_j}}_{I_{1,j}} - \underbrace{\int_{\Theta}^{\infty} \int_0^{\gamma_{\text{th}}} f_{\gamma_{M_j}, \gamma_{E_j}}(\gamma_{M_j}, \gamma_{E_j}) d\gamma_{M_j} d\gamma_{E_j}}_{I_{2,j}}. \quad (2.48)$$

By substituting (2.13) into (2.48), we get $I_{1,j} \approx I_{11,j} - I_{12,j} - I_{13,j} + I_{14,j}$ and $I_{2,j} \approx I_{21,j} - I_{22,j} - I_{23,j} + I_{24,j}$. The series-based expressions for $I_{1i,j}$, $i = \{1, 2, 3, 4\}$ can be obtained following a similar approach to (2.18)-(2.21) as

$$I_{11,j} \approx \frac{(1 + \theta)(\chi\lambda)^{m_{\gamma_{E_j}}}}{[\Gamma(m_{\gamma_{M_j}})]^2 \Gamma(m_{\gamma_{E_j}})} \sum_{\ell=0}^{\infty} \frac{(-1)^{\ell} \zeta^{\tilde{r}}}{\ell \tilde{r}} \left[\Gamma(\xi + \ell) - \sum_{\psi=0}^{\infty} \frac{(-1)^{\psi}}{\psi!(\xi + \ell + \psi)} (\zeta' \Theta)^{\xi + \ell + \psi} \right], \quad (2.49)$$

$$I_{12,j} \approx \frac{2\theta}{[\Gamma(m_{\gamma_{M_j}})]^2 \Gamma(m_{\gamma_{E_j}})} \sum_{\ell=0}^{\infty} \sum_{\psi=0}^{\infty} \frac{(-1)^{\ell}}{\ell \tilde{r}} \frac{(-1)^{\psi}}{\psi!(\tilde{a} + \psi)} (\zeta)^{\tilde{a} + \psi} \left[\Gamma(\tilde{b} + \psi) - \sum_{\varphi=0}^{\infty} \frac{(-1)^{\varphi} (\chi\Theta)^{\tilde{b} + \psi + \varphi}}{\varphi!(\tilde{b} + \psi + \varphi)} \right], \quad (2.50)$$

$$I_{13,j} \approx \frac{2\theta}{[\Gamma(m_{\gamma_{M_j}})]^2 [\Gamma(m_{\gamma_{E_j}})]^2} \sum_{\ell=0}^{\infty} \sum_{\psi=0}^{\infty} \frac{(-1)^{\ell} \zeta^{\tilde{r}}}{\ell \tilde{r}} \chi^{\tilde{t}} \frac{(-1)^{\psi}}{\psi! \tilde{s}} \left[\Gamma(\xi + \ell) - \sum_{\varphi=0}^{\infty} \frac{(-1)^{\varphi} (\chi\Theta)^{\xi + \ell + \varphi}}{\varphi!(\xi + \ell + \varphi)} \right], \quad (2.51)$$

$$I_{14,j} \approx \frac{2\theta}{[\Gamma(m_{\gamma_{M_j}})]^3 [\Gamma(m_{\gamma_{E_j}})]^2} \sum_{\ell=0}^{\infty} \sum_{\psi=0}^{\infty} \sum_{\varphi=0}^{\infty} \frac{(-1)^\ell}{\ell! \tilde{r}} \frac{(-1)^\psi}{\psi! \tilde{s}} \\ \times (\zeta)^{\tilde{a}+\varphi} \frac{(-1)^\varphi \chi^{\tilde{t}}}{\varphi! (\tilde{a} + \varphi)} \left[\Gamma(\kappa + \varphi + \omega) - \sum_{\omega=0}^{\infty} \frac{(-1)^\omega (\chi\Theta)^{\kappa+\varphi+\omega}}{\omega! (\kappa + \varphi + \omega)} \right]. \quad (2.52)$$

Likewise, the series-based expressions for $I_{2i,j}$, $i = \{1, 2, 3, 4\}$ is obtained as

$$I_{21,j} \approx \frac{1 + \theta}{[\Gamma(m_{\gamma_{M_j}})]^2 \Gamma(m_{\gamma_{E_j}})} \sum_{\ell=0}^{\infty} \frac{(-1)^\ell (\zeta' \gamma_{th})^{\tilde{r}}}{\ell! \tilde{r}} \left[\Gamma(m_{\gamma_{E_j}}) - \sum_{\psi=0}^{\infty} \frac{(-1)^\psi}{\psi! \tilde{s}} (\chi\Theta)^{\tilde{s}} \right], \quad (2.53)$$

$$I_{22,j} \approx \frac{(2\theta) \chi^{(m_{\gamma_{E_j}})}}{[\Gamma(m_{\gamma_{M_j}})]^3 [\Gamma(m_{\gamma_{E_j}})]^2} \sum_{\ell=0}^{\infty} \sum_{\psi=0}^{\infty} \frac{(-1)^\ell}{\ell! \tilde{r}} \frac{(-1)^\psi}{\psi! (\tilde{a} + \psi)} \\ \times (\zeta' \gamma_{th})^{\tilde{a}+\psi} \left[\Gamma(m_{\gamma_{E_j}}) - \sum_{\varphi=0}^{\infty} \frac{(-1)^\varphi}{\varphi! (m_{\gamma_{E_j}} + \varphi)} (\chi\Theta)^{m_{\gamma_{E_j}}+\varphi} \right], \quad (2.54)$$

$$I_{23,j} \approx \frac{2\theta}{[\Gamma(m_{\gamma_{M_j}})]^2 [\Gamma(m_{\gamma_{E_j}})]^3} \sum_{\ell=0}^{\infty} \sum_{\psi=0}^{\infty} \frac{(-1)^\ell}{\ell! \tilde{r}} \frac{(-1)^\psi}{\psi! \tilde{s}} (\zeta' \gamma_{th})^{\tilde{r}} \left[\Gamma(\tilde{t}) - \sum_{\varphi=0}^{\infty} \frac{(-1)^\varphi}{\varphi! (\tilde{t} + \varphi)} (\chi\Theta)^{\tilde{t}+\varphi} \right], \quad (2.55)$$

$$I_{24,j} \approx \frac{2\theta}{[\Gamma(m_{\gamma_{M_j}})]^3 [\Gamma(m_{\gamma_{E_j}})]^3} \sum_{\ell=0}^{\infty} \sum_{\psi=0}^{\infty} \sum_{\varphi=0}^{\infty} \frac{(-1)^\ell}{\ell! \tilde{r}} \frac{(-1)^\psi}{\psi! \tilde{s}} \\ \times \frac{(-1)^\varphi (\zeta' \gamma_{th})^{\tilde{a}+\varphi}}{\varphi! (\tilde{a} + \varphi)} \left[\Gamma(2m_{\gamma_{M_j}} + \psi) - \sum_{\omega=0}^{\infty} \frac{(-1)^\omega}{\omega! \tilde{q}} (\chi\Theta)^{\tilde{q}} \right]. \quad (2.56)$$

In (2.56), $\tilde{q} = (2m_{\gamma_{M_j}} + 2m_{\gamma_{E_j}} + \psi + \omega)$. Finally, substituting (2.49)-(2.56) into (2.42), the series-based expressions of *CSOP* can be obtained.

Now, to maximize the secrecy throughput, we find the optimal value of γ_{th} for the considered optimization problem as follows:

$$\arg \max_{\gamma_{th}} \mathcal{P}_{tx}(\gamma_{th}) \mathcal{R}_{th}, \quad (2.57a)$$

$$\text{s.t. } CSOP(\gamma_{th}, \mathcal{R}_{th}) \leq \varrho, \quad \mathcal{P}_{tx}(\gamma_{th}) \geq \nu, \quad (2.57b)$$

$$\gamma_{th} \geq \exp(\mathcal{R}_{th}) - 1, \quad \mathcal{R}_{th} > 0. \quad (2.57c)$$

In (2.57a), $\mathcal{P}_{tx}$ and *CSOP* are the function of γ_{th} and $\mathcal{R}_{th}$. Therefore, using (2.45) the possible range of γ_{th} is obtained by following $\mathcal{P}_{tx}(\gamma_{th}) \geq \nu$. Thus, $\gamma_{th} \in [\exp(\mathcal{R}_{th}) - 1, \gamma_{th,u}]$, where $\gamma_{th,u}$ is the upper limit of γ_{th} computed from (2.45). Herein, to maximize the secrecy throughput the smallest value of γ_{th} is considered whilst satisfying security constraints. Since, (2.42) is a decreasing function of γ_{th} , the optimal γ_{th} can be obtained using $CSOP(\gamma_{th}, \mathcal{R}_{th}) = \varrho$, by following $CSOP(\gamma_{th}, \mathcal{R}_{th}) \leq \varrho$, as

$$\gamma_{th} = \begin{cases} \exp(\mathcal{R}_{th}) - 1, & \text{if } \mathcal{R}_{th} \leq \ln \frac{\bar{\gamma}_M \varrho}{\bar{\gamma}_E (1 - \varrho)}, \\ \gamma_{th,u'}, & \text{otherwise.} \end{cases} \quad (2.58)$$

In (2.58), $\gamma_{th,u'}$ represents the upper limit of γ_{th} . It should be noted that the optimal value of γ_{th} given by (2.42) and (2.45) is calculated using MATLAB software for mathematical tractability. The algorithm to compute the optimal value of γ_{th} that maximizes the secrecy throughput is summarized as the algorithm 2.4.1.

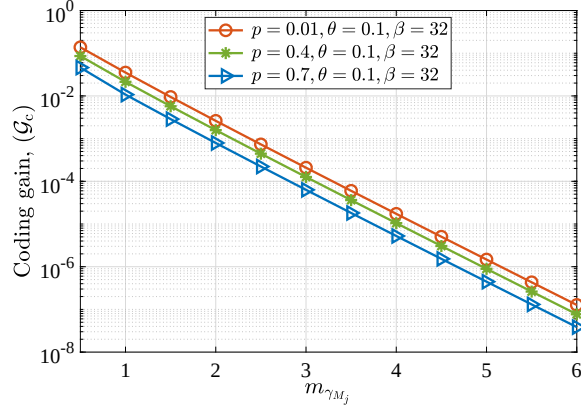


Figure 2.3: Plot of coding gain versus $m_{\gamma_{M_j}}$.

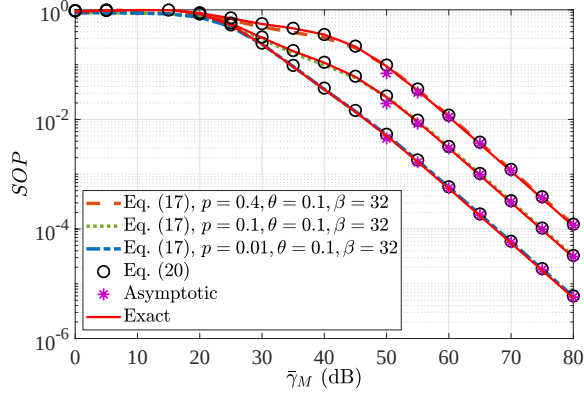


Figure 2.4: Comparison of SOP versus $\bar{\gamma}_M$ for various values of p , when $\theta = 0.1$, $\beta = 32$, and $K = 100$.

Algorithm 2.4.1: Algorithm for finding the optimal γ_{th} to maximize the secrecy throughput

- 1 *Initialization:* $\mathcal{R}_{th}$, ϱ , and ν ;
 - 2 Optimization problem given in (2.44) and obtain $CSOP$ and $\mathcal{P}_{tx}$ given in (2.42) and (2.45), respectively;
 - 3 To maximize τ , we aim to find the optimal value of γ_{th} for the considered design problem given in (2.57a) to (2.57c);
 - 4 **do**
 - 5 | Calculate γ_{th} by (2.45)
 - 6 **while** $\mathcal{P}_{tx}(\gamma_{th}) \geq \nu$, $\gamma_{th} \geq \exp(\mathcal{R}_{th}) - 1$, and $\mathcal{R}_{th} > 0$;
 - 7 Find optimal value of γ_{th} ;
 - 8 **do**
 - 9 | Calculate γ_{th} using expression $CSOP(\gamma_{th}, \mathcal{R}_{th}) = \varrho$
 - 10 **while** $CSOP(\gamma_{th}, \mathcal{R}_{th}) \leq \varrho$;
 - 11 Finally, a feasible range of optimal γ_{th} is given by (2.58);
-

2.5 Numerical Results And Discussion

In this section, numerical results and discussion are presented to analyze the performance of the proposed system. The analytical plots are obtained using the derived theoretical results. We

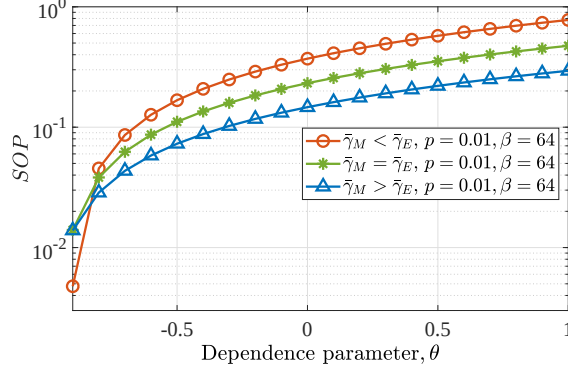


Figure 2.5: SOP versus θ for different $\bar{\gamma}_M$ and $\bar{\gamma}_E$, when $p = 0.01$ and $\beta = 64$.

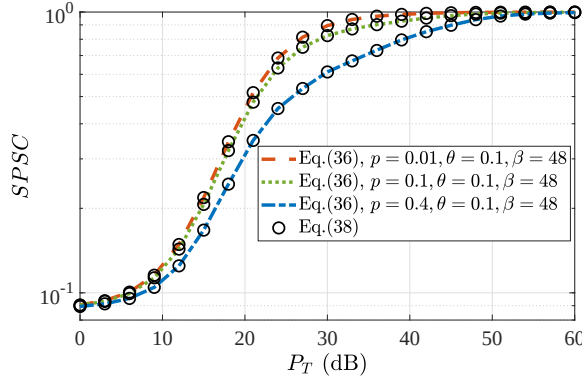


Figure 2.6: $SPSC$ versus P_T for various values of p , when $\theta = 0.1$, $\beta = 48$, and $K = 100$.

assume that the frequency is 1 MHz and the distance from A to M and from A to E is set to 300 m. Moreover, $a_0 = 8.40 \times 10^{-3} \text{ m}^{-1}$, $a_1 = 3 \times 10^{-9} \text{ s/m}$, and $c = 1$ [Zimmermann and Dostert, 2002b].

In Fig. 2.3, we plot the coding gain as a function of the diversity order for various values of p , when $\theta = 0.1$, $\beta = 32$, $K = 100$, and the transmitted power on the A to M and the A to E links are 20 dB and 10 dB, respectively. It can be observed from the figure that as the secrecy diversity order $m_{\gamma_{M_j}}$ increases, the overall $\mathcal{G}_{c,j}$ decreases. Moreover, the $\mathcal{G}_{c,j}$ improves as the value of p is reduced from 0.7 to 0.01.

Figure 2.4 illustrates the SOP versus $\bar{\gamma}_M$ curves for various values of p , when $\theta = 0.1$, $\beta = 32$, $K = 100$, and transmitted power on the A to E link is fixed at 20 dB. It is explicitly shown in the figure that the SOP performance is better for lower values of p and increasing p degrades the SOP . This is because the frequent occurrence of impulsive noise causes, the PLC system's overall noise power to increase, which negatively affects the SOP performance of the considered system. Moreover, it is observed that the lower bound SOP curves are very close to the exact SOP curves, which is the accuracy of the derived SOP lower bound. Additionally, the asymptotic SOP as a function of the average SNR is shown in Fig. 2.4. Here, the SOP is 5.97×10^{-5} and 5.97×10^{-6} at 70 dB and 80 dB, respectively, for $p = 0.01$, $\theta = 0.1$, $\beta = 32$, and the transmitted power on the A to E link is fixed at 20 dB. Therefore, the asymptotic slope is computed as $\log_{10}(5.97 \times 10^{-5}) - \log_{10}(5.97 \times 10^{-6}) = 1 = \min\{m_{\gamma_{M_j}}, 2m_{\gamma_{M_j}}\}$, which is in line with the asymptotic slope theoretically calculated in (2.32).

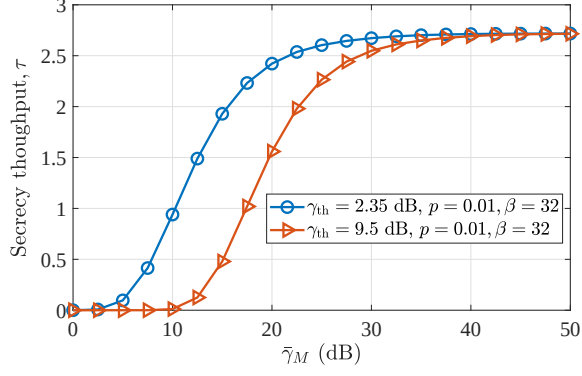


Figure 2.7: Secrecy throughput versus $\bar{\gamma}_M$ for various values of γ_{th} , when $p = 0.01$, $\beta = 32$, and $K = 100$.

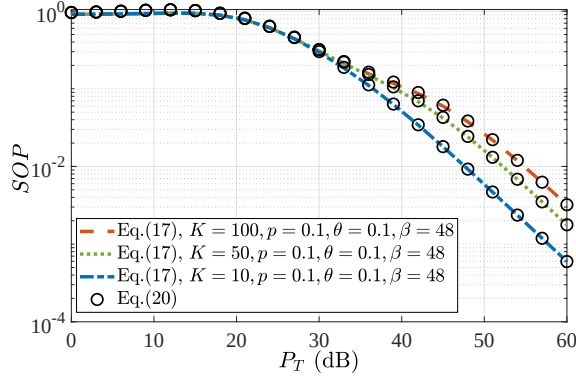


Figure 2.8: SOP versus P_T plot for different values of K , when $p = 0.1$, $\theta = 0.1$, and $\beta = 48$.

In Fig. 2.5, the SOP is plotted as a function of the dependence parameter θ for $p = 0.01$ and $\beta = 64$ under different SNR conditions ($\bar{\gamma}_M > \bar{\gamma}_E$, $\bar{\gamma}_M = \bar{\gamma}_E$, and $\bar{\gamma}_M < \bar{\gamma}_E$). When $\bar{\gamma}_M < \bar{\gamma}_E$, the SOP performance deteriorates with increasing θ compared to the other two conditions. Receiving a high SNR at E increases its channel capacity, which implies higher chances of secrecy outage. Therefore, the PLC channel becomes less secure. Similarly, when $\bar{\gamma}_M > \bar{\gamma}_E$ the SOP performance is improved. This is because receiving a high SNR at M increases the capacity of the main PLC channel. As a result, there are less chance of an outage. Finally, for $\bar{\gamma}_M = \bar{\gamma}_E$ the SOP performance is moderate. In all the three conditions, it is revealed that as θ increases to 1, the SOP also tends to 1. This observation is quite intuitive because high positive values of θ indicate higher dependency between the main channel and the eavesdropper channel, thereby making the PLC transmission less secure.

Figure 2.6 shows the $SPSC$ versus the transmitted power on the A to M link for various values of p , when $\theta = 0.1$, $\beta = 48$, and $K = 100$, under a fixed transmitted power of 20 dB on the A to E link. It is clearly shown in the figure that the $SPSC$ performance improves as the value of p is reduced from 0.4 to 0.05.

Figure 2.7 depicts the secrecy throughput versus $\bar{\gamma}_M$ for different values of γ_{th} , when $p = 0.01$, $\beta = 32$, $K = 100$, $\mathcal{R}_{th} = 1$, and $\bar{\gamma}_E = 0$ dB. Herein, $\gamma_{th} = 2.35$ dB is obtained by $\gamma_{th} = \exp(\mathcal{R}_{th}) - 1$. Likewise, $\gamma_{th} = 9.5$ dB is computed assuming $CSOP(\gamma_{th}, \mathcal{R}_{th}) = \varrho$ for $p = 0.01$, $\beta = 32$, $\varrho = 0.2$, and $\theta = 0.1$, when the transmitted power on the A to M and the A to E links fixed at 20 dB and 8 dB, respectively. It is observed that secrecy throughput improves with a lower value of γ_{th} because there are less chances of outage for lower values of γ_{th} . Similarly, there is a high

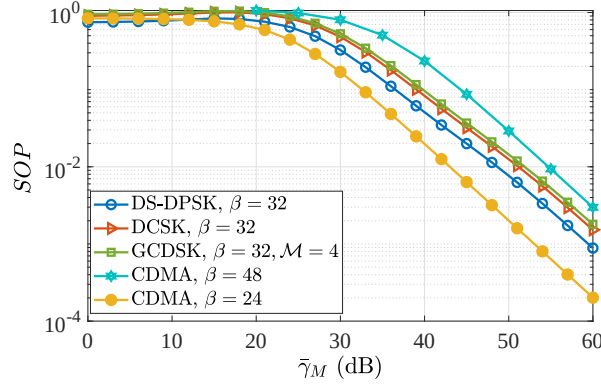


Figure 2.9: SOP versus $\bar{\gamma}_M$ for different modulation schemes, when $\beta = 24, 32, 48$, and $\mathcal{M} = 4$.

outage probability for high values of γ_{th} , and therefore the secrecy throughput decreases with the increase of γ_{th} . It is noted that, the computed values of γ_{th} that maximize the secrecy throughput satisfy all the optimization conditions given in (2.57b) and (2.57c). For instance, it is observed from the figure that when $\gamma_{th} = 9.5$ dB, $CSOP = 0.134$ and $\mathcal{P}_{tx} = 0.987$ for $\varrho = 0.2$ and $\nu = 0.4$, which satisfy the optimization conditions in (2.57b) and (2.57c). Additionally, it is also revealed that the secrecy throughput saturates beyond 40 dB average SNR. This is due to the fact that all communication systems have finite limits on bandwidth, modulation schemes, coding rates, and hardware limitations. Once these aforementioned parameters reach their limits, no further secrecy throughput can be achieved as the average SNR increases.

The SOP of the considered PLC system as a function of transmitted power on the A to M link for different values of K is shown in Fig. 2.8. The curves are plotted for $p = 0.1, \theta = 0.1, \beta = 48$, and the transmitted power of the A to E link is fixed at 20 dB. It is observed that the SOP performance degrades as the value of K increases. Moreover, it is inferred from the plot that the value of the impulsive noise index parameter severely affect the behavior of the proposed PLC communication system.

Figure 2.9 shows the comparison of SOP as a function of $\bar{\gamma}_M$ for the different modulation schemes such as DCSK, DS-DPSK, GCDSK, and CDMA. It is found that the SOP performance of DCSK is superior to GCDSK for selected values of β and delay block ($\mathcal{M}$). Additionally, the GCDSK system structure is complex and complicated. It requires an additional delay, correlator, and multiplier [Zheng *et al.*, 2017]. However, CDMA performs worse with a higher value of β and vice-versa. Further, it is noticed that the SOP performance of DS-DPSK is better than DCSK. However, the DCSK system inherits chaos signal properties that make the interception and decoding of data more difficult for an eavesdropper.

2.6 Summary

In this chapter, we investigated the secrecy performance of DCSK-based PLC systems under correlated Log-normally distributed PLC wiretap channel. The FGM Copula approach facilitated the statistical characterization in terms of PDF and mathematical modeling of the PLS performance metrics in terms of SOP and SPSC. Specifically, the FGM Copula explicitly shows the correlation between the main and the wiretap channels in terms of the dependence parameter. To get further insights into the SOP study, we performed the asymptotic analysis of SOP. We also presented an algorithm to maximize the secrecy throughput under SOP constraints. The algorithm is used to find the optimal SNR for which secrecy throughput is maximum. We examined the influence of

various system parameters using numerical results to get more insights into the considered PLC system from the standpoint of engineering design. The key findings of this chapter are summarized as follows:

- The SOP performance always improves with an increasing value of the average SNR of the main channel and when the average SNR of the main channel is greater than the average SNR of the eavesdropper channel.
- SOP performance deteriorates with an increasing value of θ . This observation is quite intuitive because high positive values of θ indicate higher dependency between the main channel and the eavesdropper channel, thereby making the PLC transmission less secure.
- It is also noticed that the SOP performance degrades with a higher occurring probability of impulsive noise. It is evident that the frequent occurrence of impulsive noise is more harmful to the overall performance of the PLC system.
- In the asymptotic study, a relation was established between the secrecy coding gain and the secrecy diversity order. It is shown that the secrecy diversity order depends on the shaping parameter (m_{γ_M}) of the main channel.
- It is noted that the secrecy throughput improves with a lower value of optimal SNR and decreases with a higher value of optimal SNR.
- The SPSC performance is better with an increasing value of transmitted power and decreases with a higher value of the arrival probability of the impulsive noise component.
- Moreover, we compare the SOP performance of the proposed system with GCDSK, DS-DPSK, and CDMA. It is found that the proposed system performs better than GCDSK and CDMA for certain values of β .

